PACIFIC JOURNAL OF MATHEMATICS
Vol. 91, No. 1, 1980

DIRECT FACTORIZATIONS OF MEASURES

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In this paper we want to investigate the question, to what extent can the disintegration of some measure on an arbitrary Suslin space with respect to some measurable function f be replaced by the image measure under some function g inverting f, such that the "outcome" of the situation under a function h is not changed. Such a direct factorization, as we call it, is modulo some conditions about atoms of the measures in general only possible, if the range of h is countable. But there are always solutions to the problem in a weak sense. The results have applications in game theory to the problem of "elimination of randomization".

Our staring point are some results about the compactness and convexity of the range of some measure operations. They are closely related to Lyapunov's theorem [10].

In § 2 we recall some known results about the disintegration of measures on Suslin spaces.

The problem of direct factorizations of measures is made precise in § 3 and solved there for the case where the "outcome"-set C is countable. Of course, some restrictions concerning the atoms of the measures are necessary. A counterexample shows that this result cannot be generalized to compact metrizable C. Thus we introduce in § 4 the notion of a weak direct factorization and show that such a weak direct factorization exists even if C is an arbitrary Suslin space.

It is quite obvious that the solutions to the direct factorization problem are extreme points in a certain convex space of measures on a Suslin set. In fact, we show in § 5 that if C is countable or if we regard the weak problem, the extreme points of this set are exactly the solutions to the corresponding factorization problem. Under somewhat different situations such characterizations have been found in [5].

As mentioned at the beginning, we shall apply the results to a question in game theory in § 6. The application shows, when random strategies (=behavior strategies) can be equivalently replaced by nonrandom ones. Such questions of "elimination of randomization" have been treated in [3] and [4] in the finite case and are here generalized to arbitrary Suslin spaces.

1. Convex ranges by nonatomic measures. Since in later sections we are interested when some integral operators have com-