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INTEGRALLY CLOSED IDEALS AND ASYMPTOTIC PRIME DIVISORS

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The first theorem characterizes local (Noetherian) domains that have a height one maximal ideal in their integral closure as those local domains whose maximal ideal M is a prime divisor (= associated prime) of the integral closure I_a of all nonzero ideals I contained in large powers of M. The second theorem describes (modulo a mild assumption) all local domains R that have the following property: for each ideal I in R and for all large n, all the ideals I^n and $(I^n)_a$ have the same prime divisors.

1. Introduction. In [12, (9)], local domains that have a depth one prime divisor of zero in their completion were characterized as those local domains whose maximal ideal M is a prime divisor of all nonzero ideals I contained in large powers of M. Also, a similar characterization of a subclass of the class of local domains whose completions have a depth one *minimal* prime ideal was given in [12, (11)], but I was unable to give an analogous characterization of this entire class in [12]. The first of the main results in this paper, Theorem 1, gives the desired characterization, which was described in the abstruct. This result is actually proved for local *rings*, and the result [12, (9)] is also extended to this case (in Theorem 0).

As an application of this result, we consider, in §3, the following class of local domains R: for each ideal I in R and for all large n, all the ideals I^n and $(I^n)_a$ have the same prime divisors. Concerning this class, McAdam and Eakin showed in [4, Prop. 24] that for each ideal I in a Noetherian UFD of altitude two and for all $n \geqq 1$, all the ideals I^n and $(I^n)_a$ have the same prime divisors. On weakening the conclusion to "for all large n," McAdam showed in [5, Thm. 6 and Prop. 8] that this continues to hold for all integrally closed Noetherian domains of altitude two and "integrally closed" is a partly necessary hypothesis. In [13, Prop. 12], I added two other types of local domains that have this latter property, and the second of the main results in this paper, Theorem 4, describes all Noetherian domains that have this property (assuming that integrally closed local domains of altitude three are catenary). The proof of Theorem 4 requires consideration of several cases, and Theorem 1 plays an important role in some of these cases.

Concerning this second class of rings, I am indebted to S.