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IRREDUCIBLE OPERATORS WHOSE SPECTRA ARE SPECTRAL SETS

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In this note those compact subsets of the plane that are the spectra of irreducible subnormal operators are characterized.

In December 1977 John B. Conway presented a colloquium talk at Virginia Tech and asked, "Which compact subsets of the plane are the spectra of irreducible subnormal operators?" If the adjective irreducible is replaced by pure in this question, then Clancey and Putnam [2] have the following answer: A compact set K is the spectrum of a pure subnormal operator if and only if for every open disc Δ that has a nonempty intersection with K, we have $R(K \cap \Delta^-) \neq C(K \cap \Delta^-)$. Similarly, our answer to Conway's question will be a function algebraic characterization. For the basic facts concerning this area we refer to [3] or [10].

If $\mathscr{H}$ is a separable Hilbert space the algebra of continuous operators on $\mathscr{H}$ will be denoted $\mathscr{B}(\mathscr{H})$. An operator $T \in \mathscr{B}(\mathscr{H})$ is irreducible if $\mathscr{H}$ has no nonzero subspace that is invariant under T and its adjoint T^*. For the basic facts concerning subnormal operators we refer to [4]. If $T \in \mathscr{B}(\mathscr{H})$ then $\sigma(T)$ will denote the spectrum of T.

If K is a compact subset of the plane, $\mathscr{R}(K)$ will denote the collection of rational functions with poles off K; the uniform closure of $\mathscr{R}(K)$ in $C(K)$, the algebra of continuous functions on K, is denoted $R(K)$. If φ belongs to the maximal ideal space of $R(K)$, then there exists a point $z \in K$ such that $\varphi(f) = f(z)$ for all $f \in R(K)$. Hence the Gleason parts of $R(K)$ form a partition of K.

If $T \in \mathscr{B}(\mathscr{H})$ and K is a compact set containing $\sigma(T)$, then K is called a spectral set for T if

$$\text{(i)} \qquad \|f(T)\| \leqq \|f\|_K$$

for all $f \in \mathscr{R}(K)$. (Here $\|f\|_K$ denotes the sup norm of f on K). If K is a spectral set for T, it is easy to define $f(T)(\in \mathscr{B}(\mathscr{H}))$ for all $f \in R(K)$. A basic fact about a subnormal operator S is that $\sigma(S)$ is a spectral set and equality holds in (i) with $K = \sigma(S)$ for all $f \in R(\sigma(S))$.

THEOREM 1. *A compact set K is the spectrum of an irreducible operator T whose spectrum is a spectral set if and only if $R(K)$ has one nontrivial Gleason part G and $G^- = K$.*