PACIFIC JOURNAL OF MATHEMATICS
Vol. 91, No. 2, 1980

TOPOLOGICAL ALGEBRAS WITH ORTHOGONAL SCHAUDER BASES

TAQDIR HUSAIN AND SALEEM WATSON

Topological algebras with Schauder orthogonal bases are studied. Radicals, closed ideals and closed maximal ideals of such algebras are described. It turns out that a locally m-convex algebra with identity and having an orthogonal basis is metrizable. This implies that a complete locally m-convex algebra with an orthogonal basis and identity is algebraically and topologically isomorphic with the Fréchet algebra of all complex sequences.

Introduction. Let A be a topological algebra. A (Schauder) basis $\{x_n\}$ in A is called an *orthogonal* (*Schauder*) *basis* if $x_nx_m = \delta_{nm}x_n$, $n, m = 1, 2, \cdots$ where δ_{nm} denotes the Kronecker delta. Algebras with such bases (actually a variation of this definition which we will discuss below) were first studied by Husain. In [3] Husain and Liang proved that every multiplicative linear functional on a Fréchet algebra (i.e., complete metrizable locally m-convex algebra) with an unconditional orthogonal basis is continuous. This result answers Michael's question [5] (as to whether every multiplicative linear functional on a Fréchet algebra is continuous) in the affirmative for such Fréchet algebras.

In this paper we study the structure of topological algebras having an orthogonal Schauder basis. In § 1 we discuss some properties of bases in topological algebras which we will use later. In § 2 we describe the closed ideals and show that each closed ideal is the closure of the linear span of the basis elements it contains. In § 3 we give a characterization of complete locally m-convex algebras with identity having an orthogonal basis. In another paper [4] we study topological algebras having unconditional orthogonal bases.

For definitions and results concerning bases in Banach spaces see [1], [7]. For general notions regarding topological algebras see Michael [5] and Zelazko [8]. A sequence $\{x_n\}$ in a topological vector space E is a basis if for each $x \in E$ there is a unique sequence of scalars $\{\alpha_n\}$ such that $x = \sum_{n=1}^{\infty} \alpha_n x_n$. Each linear functional $x_n^*(x) = \alpha_n$ is called a coefficient functional. If each x_n^* is continuous then $\{x_n\}$ is called a Schauder basis. It is well known that each basis in a complete metrizable vector space is a Schauder basis. We show that each orthogonal basis in a locally m-convex algebra is a Schauder basis (Prop. 3.1) and each unital locally m-convex algebra A with an orthogonal basis is metrizable (Theorem 3.3) and if, in addition