

ON THE LATTICE OF VARIETIES OF BANDS OF GROUPS

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In this paper we prove that the lattice of varieties of bands of groups is modular and apply this to direct decompositions of various sublattices. The join of the varieties of bands and of completely simple semigroup is shown to be the variety of "pseudo-orthodox" bands of groups.

1. **Introduction.** When considered as semigroups with an additional unary operation $x \rightarrow x^{-1}$, where x^{-1} denotes the (unique) inverse of x in the subgroup to which it belongs, the class CR of *completely regular* semigroups (often called *unions of groups*) forms a variety of universal algebras, containing as a subvariety the variety BG of *bands of groups* (those completely regular semigroups on which \mathcal{H} is a congruence) ([12]). In this paper results of Spitznagel [14] on the lattice of congruences on a band of groups are applied to show that $\mathcal{V}(BG)$, the lattice of subvarieties of BG , is modular (Theorem 3.1). Petrich [12, 13] considered various subvarieties of BG but left open the problem [13, p.1196] of finding the join of the subvarieties B and CS (of bands and of completely simple semigroups respectively). We show that $B \vee CS = POBG$, the variety of *pseudo-orthodox* bands of groups, and is thus strictly contained in BG . (If V is a variety of completely regular semigroups and $S \in CR$ we shall call S *pseudo- V* if $eSe \in V$ for every idempotent e of S .) This result is actually an immediate corollary to our characterization of the join $O \vee NBG$ of the varieties of orthodox completely regular semigroups and of normal bands of groups. Theorem 3.1 is also applied to directly decompose various sublattices of $\mathcal{V}(BG)$.

2. **Preliminaries.** For background to this paper the reader is referred to [13] where defining identities are presented for most of the varieties encountered here. Various subvarieties of CR are shown on the diagram on p. 1172 of [13]. For easy reference we will give a list of our abbreviations for these:

- CR : completely regular semigroups
- BG : bands of groups
- NBG : normal bands of groups
- OBG : orthodox bands of groups
- B : bands
- CS : completely simple semigroups
- NB : normal bands