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# ON THE HOMOMORPHIC AND ISOMORPHIC EMBEDDINGS OF A SEMIFLOW INTO A RADIAL FLOW

MICHAEL EDELSTEIN

**It is the main purpose of this paper to prove the following two theorems.**

THEOREM I. **(Isomorphism) Let $(X, \boldsymbol{R}^+, f)$ be a semiflow on a separable metric space $(X, d)$, having the properties:**

**(i) there is an $\omega \in X$ such that, for each neighborhood $U$ of $\omega$, there is a $T \in \boldsymbol{R}^+$ with $f[X, t] \subset U$ for all $t \geqq T$;**

**(ii) for each $t \in \boldsymbol{R}^+, f(\cdot, t)$ is a homeomorphism of $X$ onto a closed subspace of $X$.**

**Then $(X, \boldsymbol{R}^+, f)$ is isomorphic to a radial semiflow on a subset of the Hilbert Cube in $l^2$.**

THEOREM II. **(Homomorphism) If $(X, \boldsymbol{R}^+, f)$ satisfies the hypotheses of Theorem I, with (i) replaced by**

**(i′) $\cap \{f[X, t]: t \geqq 0\} = \{\omega\}$ for some $\omega \in X$, then $\{X, \boldsymbol{R}^+, f)$ is homomorphic to a radial semiflow on a subset of the Hilbert Cube $C$ and the subsemiflow induced on $X/\{\omega\}$ is isomorphic to a radial semiflow in $C$.**

## 1. Introduction.

Let $X$ be a nonempty subset of a normed linear space and suppose that, with $0 < \lambda < 1$,

$$f(x, t) = \lambda^t x \quad ((x, t) \in X \times \boldsymbol{R}) . \tag{1}$$

The triple $(X, \boldsymbol{R}, f)$, with $f$ as above, determines a dynamical system or a flow (cf. [4], [5]) on $X$ such that the semitrajectory of each $x \neq 0$ is a line segment joining $x$ with the origin 0. The terms "a radial flow" or "a radial dynamical system" seem appropriate. Similarly, with $\boldsymbol{R}$ replaced by $\boldsymbol{R}^+$, the nonnegative reals, we refer to $(X, \boldsymbol{R}^+, f)$ as a radial semiflow, or a radial semidynamical system. By a homomorphic (isomorphic) embedding of $(X, \boldsymbol{R}, f)$ into $(Y, \boldsymbol{R}, g)$ we understand a one-one continuous mapping (a homeomorphism) $h$ of $X$ into $Y$ such that

$$h(f(x, t)) = g(h(x), t) \ ((x, t) \in X \times \boldsymbol{R}) .$$

A similar definition applies to semiflows. Thus, to say that $(X, \boldsymbol{R}^+, f)$ is isomorphic to a radial semiflow means that a homeomorphism $h$ of $X$ into a normed linear space exists such that

$$h(f(x, t)) = \lambda^t h(x) \ ((x, t) \in X \times \boldsymbol{R}^+) . \tag{2}$$

In a recent paper L. Janos [3] proved that a semiflow $(X, \boldsymbol{R}^+, f)$