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ON THE CLOSED IDEALS IN $A(W)$

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This paper is about the ideal theory of the algebra of functions continuous on the closure and holomorphic in the interior of a domain on a compact Riemann surface. The description of the closed ideals in the disc algebra is shown to apply to an ideal whose hull meets the boundary of the domain in a finite union of analytic arcs. The canonical factorization into inner and outer functions in the disc is replaced by a potential theoretic decomposition theorem, thus allowing essentially the same description to be carried over. The basically local nature of the problem is used to reduce it to the previously known ideal theory of a compact bordered Riemann surface. This reduction is facilitated by a factorization theorem that is proved by potential theoretic methods.

Let W be a domain (i.e., open, connected set) on a compact Riemann surface S, let ∂W denote the boundary of W and $\bar{W} = W \cup \partial W$ its closure. Let $A(W)$ be the set of all complex valued functions that are continuous on $\bar{W}$ and holomorphic on W; $A(W)$ is a Banach algebra in the uniform norm. In the case of the unit disc, Beurling (unpublished), and Rudin [8] described the closed ideals of $A(W)$; see also [5] for an exposition of these results.

In the case of a finite Riemann surface, Voichick [10] found an analogous description. This case was also treated by Hasumi [3], and Stanton [9]. These descriptions are essentially local; thus one may ask if they extend in some form to more general domains. In this paper, we obtain corresponding results for closed ideals of a certain type. We assume that ∂W contains a subset Γ such that $W \cup \Gamma$ is a bordered Riemann surface with analytic border Γ, that W lies on one side of Γ, and that Γ has finitely many components. We describe those closed ideals in $A(W)$ whose hulls lie in $W \cup \Gamma$. We reduce our problem to the ideal theory of a finite Riemann surface by means of a factorization theorem which allows us to separate the singularities of functions in $A(W)$. The factorization theorem follows from the decomposition theorem of Parreau [6], so our methods are somewhat potential theoretic.

In § 1, we illustrate our methods in the case of an annulus in the complex plane. In § 2 we collect some facts about harmonic functions necessary for the proof of the factorization theorem in § 3 and the description of closed ideals in § 4.

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