PACIFIC JOURNAL OF MATHEMATICS
Vol. 92, No. 1, 1981

CORRECTION TO "PEANO MODELS WITH MANY GENERIC CLASSES"

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A theorem in the paper referred to in the title had a faulty proof which is repaired herein by employing the Halpern Läuchli-Laver-Pincus Partition Theorem.

In [4] a generalization of the MacDowell-Specker Theorem on extensions of models of Peano arithmetic was stated as Theorem 3.1. Unfortunately, the proof presented there turned out to be faulty. The purpose of this note is, first, to repair that proof, and, second, to present a slight strengthening of the theorem.

All unexplained notation and terminology is taken from [4]. If $\mathscr{N} \subseteq \mathscr{M}$ are such that whenever $X \subseteq M$ is definable in $\mathscr{M}$, then $X \cap N$ is definable in $\mathscr{N}$, then we say that the extension is *conservative.*

THEOREM. *Let $\mathscr{N}$ be a model of Peano arithmetic and $\langle X_i: i \in I\rangle$ be a collection of mutually generic classes of $\mathscr{N}$. Then $(\mathscr{N}, \langle X_i: i \in I\rangle)$ has a proper, conservative, elementary end-extension $(\mathscr{M}, \langle Y_i: i \in I\rangle)$. In addition, for each $J \subseteq I$ there is a unique $\mathscr{M}_J \preccurlyeq \mathscr{M}$ which is a proper elementary extension of $\mathscr{N}$ such that*

$$J = \{i \in I\text{: there is a class } Y \text{ of } \mathscr{M}_J \text{ such that } X_i = Y \cap N\}\ .$$

Each $(\mathscr{M}_J, \langle Y_j \cap M_J: j \in J\rangle)$ is conservative extension of $(\mathscr{N}, \langle X_j: j \in J\rangle)$.

Theorem 3.1 of [4] consists just of the first two sentences of the above theorem. This will be proved first. The latter two sentences make up the strengthening which will be established by noting the appropriate changes to make in the first proof.

The error in [4] was that a too weak combinatorial theorem was used, the correct theorem being the Halpern-Läuchli-Laver-Pincus partition theorem for trees. Proofs of this theorem can be found in [3] or [2], these proofs being easily formalizable in Peano arithmetic. Harrington has a slick proof which, while apparently not formalizable in Peano arithmetic, nevertheless, can be applied in all countable models of Peano arithmetic, and thus shows that the theorem actually is a consequence of the axioms of Peano arithmetic.

Some definitions are necessary in order to state this theorem in a form which best suits our purposes.

Let $\trianglelefteq$ be the binary relation on ω which is defined by:

$$x \trianglelefteq y \longleftrightarrow \exists z[(x+1)2^z \leqq y+1 < (x+2)2^z]\ .$$