PACIFIC JOURNAL OF MATHEMATICS
Vol. 92, No. 1, 1981

TAUBERIAN THEOREMS FOR MATRICES GENERATED BY ANALYTIC FUNCTIONS

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Several classes of summability matrices are determined by the coefficients of Maclaurin series of the products of certain analytic functions. These matrices include generalizations of the transforms of Lototsky, Taylor, and others. It is proved that under rather weak restrictions on the analytic functions, $x_k - x_{k+1} = o(k^{-1})$ is a Tauberian condition for the resulting matrix transformations.

1. Introduction. Several classes of summability transforms are generated by products of analytic functions. The matrix $(a_{n,k})$ of such a transform is given by

$$\prod_{k=0}^{n} f_k(z) = \sum_{k=0}^{\infty} a_{n,k} z^k \,, \tag{1}$$

where $f_k(z)$ is analytic at $z = 0$ $(k = 0, 1, 2, \cdots)$. This class of transforms includes, for example, the well-known Euler-Knopp means [6, pp. 56-60] and the Taylor transforms [6, pp. 60-64]. In addition to these two special cases, the transforms of this class for which we shall prove Tauberian theorems are the following: the Karamata transform [8, 9], the generalized Lototsky transform [4], and the $\mathscr{T}(r_n)$ transform [7]. We also give a Tauberian theorem for the $T(r_n)$ transform [5] which, although not a member of this class, is very similar to the others.

In this paper we shall state the Tauberian theorems in sequence-to-sequence form; thus, a typical Tauberian condition for a sequence x is $(\Delta x)_k = o(k^{-1})$, where Δx is given by $(\Delta x)_k = x_k - x_{k+1}$. Our proofs will use recently developed techniques [1, 2] that are based on the concept of a "block-dominated" matrix. For each n, let $\{a_{n,k}\}_{k=1+\mu(n)}^{\nu(n)}$ be a block of consecutive terms of nth row of the matrix A; then A is *dominated by the sequence of blocks* $\{a_{n,k}\}_{k=1+\mu(n)}^{\nu(n)}$ $(n = 0, 1, \cdots)$ provided that

$$\liminf_n \left\{ \left| \sum_{k=1+\mu(n)}^{\nu(n)} a_{n,k} \right| - \sum_{k \leq \mu(n)} |a_{n,k}| - \sum_{k > \nu(n)} |a_{n,k}| \right\} > 0 \,. \tag{2}$$

Then $L_n \equiv \nu(n) - \mu(n)$ is called the length of the block in the nth row. The results from [1, 2] that we shall use are stated here for convenience.

THEOREM A. *Let A be a regular matrix that is dominated by*