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TWO THEOREMS ON GENERAL SYMMETRIC SPACES

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An important result in the theory of Riemannian symmetric spaces is the theorem that the universal covering space of a complete locally symmetric space is symmetric. The proof uses the highly nontrivial property enjoyed by Riemann (but by neither Finsler nor G-) spaces that they are automatically analytic when locally symmetric and of class C^1. Our first theorem, nevertheless, extends the above result to locally symmetric G-spaces, which need not be smooth and which even when smooth are only Finsler, and not necessarily Riemann, spaces. Our second theorem states that a generic locally symmetric G-space is locally Minkowskian. This theorem has no analogue in Riemannian geometry.

1. Introduction. A basic theorem on symmetric Riemann spaces states that a complete locally symmetric space has a globally symmetric (or, shorter, symmetric) universal covering space. In [5] we attributed this result wrongly to Ambrose and Singer [1], who prove a more general fact; the mentioned theorem is due to Ehresmann [6] and Borel and Lichnérowicz [2].

One of the principal purposes of the present paper is extending the theorem to G-spoces (which are by definition complete).

THEOREM I. *A locally symmetric G-space has a globally symmetric universal covering space.*

In view of the fact that our proof uses a simple geometric idea instead of the elaborate machinery of the literature some remarks on I are in order. A Riemann space of class C^1 is a, say, n-dimensional manifold with a differentiable structure and a positive definite quadratic form $ds^2 = \sum g_{ik}(x)\xi^i\xi^k$, where $\xi = (\xi^1, \cdots, \xi^n)$ is a tangent vector and the g_{ik} are of class C^1. We choose this old fashioned definition since it facilitates explaining the difference between a Riemann and a Finsler space where $ds^2 = F^2(x, \xi)$ and F satisfies certain standard conditions [3, § 15], in particular $F(x, k\xi) = |k|\, F(x, \xi)$ for real k, $F(x, \xi) > 0$ for $\xi \neq 0$ and $F(x, \xi) = 1$ is for fixed x a strictly convex hypersurface in the tangent space instead of an ellipsoid.

A fundamental difference appears at once: in the Riemannian case ds^2 depends for fixed x analytically on ξ. This need not be so even in the simplest (symmetric) Finsler space of class C^∞, namely a Minkowski space whose spheres are of class C^∞ but not analytic.