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MAPS ON SIMPLE ALGEBRAS PRESERVING ZERO PRODUCTS. II: LIE ALGEBRAS OF LINEAR TYPE

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The study of maps on an algebra which preserve zero products is suggested by recent studies on linear transformations of various types on the space of $n \times n$ matrices over a field, particularly Watkins' work on maps preserving commuting pairs of matrices. This article generalizes the result of Watkins by determining the bijective semilinear maps f on a Lie algebra L with the property that

$$[x, y] = 0 \Longrightarrow [f(x), f(y)] = 0,$$

where $x, y \in L$, for a class of Lie algebras constructed from finite-dimensional simple associative algebras.

Introduction. In [8] we began the study of the semilinear maps on an algebra over a field k which preserve zero products, a problem arising from recent investigations characterizing the linear transformations on the $n \times n$ matrix algebra $M_n(k)$ over k which preserve various properties, particularly the work of Watkins on maps preserving commuting pairs of matrices [7]. If L is a Lie algebra, this means that we are concerned with the bijective semilinear maps f on L such that $[f(x), f(y)] = 0$ for all pairs of elements x, y of L such that $[x, y] = 0$. We say that f *preserves zero Lie products.*

If L is finite-dimensional, these maps f form a group $G(L)$ [8]. Clearly $G(L)$ contains the group G_1 of all semilinear automorphisms and anti-automorphisms (semilinear maps which are automorphisms or anti-automorphisms of the multiplicative structure of L), the group of units G_2 of the centroid of L (the algebra of linear transformations which commute with left multiplications in L), and the group G_3 of all bijective transformations f of the form $f(x) = x + g(x)$, where g is a linear map of L into its center $Z(L)$. Let $G_0(L) = G_1G_2G_3$.

In this paper we determine $G(L)$, for a class of simple Lie algebras L. These are obtained by taking finite-dimensional simple associative algebras A over a field k and forming the Lie algebra $L = [A, A]/[A, A] \cap Z(A)$, where $[A, A]$ is the subspace spanned by all the commutators $[x, y] = xy - yx$, and $Z(A)$ is the center of A. If A is noncommutative, then L is a simple Lie algebra, except when A has characteristic 2 and is 4-dimensional over $Z(A)$ [1, p. 17]. Except for cases of "small length," we show that $G(L) = G_0(L)$ for such a Lie algebra L. In fact, we can deal with a wider class of