PACIFIC JOURNAL OF MATHEMATICS
Vol. 92, No. 2, 1981

THE FACTORIZATION OF H^p ON THE SPACE OF HOMOGENEOUS TYPE

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Let K be a Calderon-Zygmund singular integral operator with smooth kernel. That is, there is an $\Omega(x)$ defined on $R^n\backslash\{0\}$ which satisfies

$$\begin{aligned} &\int_{|x|=1} \Omega = 0\,, \quad \Omega \not\equiv 0\,, \\ (*)\qquad &\Omega(rx) = \Omega(x) \quad \textbf{when} \quad r > 0 \quad \textbf{and} \quad x \in R^n\backslash\{0\}\,, \\ &|\Omega(x) - \Omega(y)| \le |x - y| \quad \textbf{when} \quad |x| = |y| = 1\,, \end{aligned}$$

and that

$$Kf(x) = P.\ V. \int_{R^n} \Omega(x-y)\,|x-y|^{-n} f(y)dy\,.$$

Let

$$K'f(x) = P.\ V. \int_{R^n} \Omega(y-x)|y-x|^{-n} f(y)dy\,.$$

R. Coifman, R. Rochberg and G. Weiss showed the weak version of the factorization theorem of $H^1(R^n)$ and that was refined by Uchiyama in the following form.

THEOREM A. *If $1 < q < \infty$ and $1/q + 1/r = 1$, then*

$$\begin{aligned} c_{K,q}\|f\|_{H^1(R^n)} \leqq \inf \Big\{\sum_{i=1}^{\infty} \|g_i\|_{L^q}\|h_i\|_{L^r}: \\ f = \sum_{i=1}^{\infty}(h_i K g_i - g_i K' h_i)\Big\} \leqq c'_{K,q}\|f\|_{H^1(R^n)}\,. \end{aligned}$$

In this note, we extend Theorem A to $H^p(X)$, where $p \in (1 - \varepsilon_X, 1]$ and X is a space of homogeneous type with certain assumptions.

1. Preliminaries. In the following, $A > 1$ and $\gamma \leqq 1$ are positive constants depending only on the space X.

Let X be a topological space endowed with a Borel measure μ and a quasi-distance d such that

$$(1)\qquad d(x, y) \geqq 0$$

$$(2)\qquad d(x, y) > 0 \quad \text{iff} \quad x \neq y$$

$$(3)\qquad d(x, y) = d(y, x)$$

$$(4)\qquad d(x, z) \leqq A(d(x, y) + d(y, z))$$