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MONODROMY AND INVARIANTS OF ELLIPTIC SURFACES

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The purpose of this research is to analyze and compute the monodromy representation of the Gauss Manin connection associated with an elliptic surface and to relate properties of the monodromy to geometric properties of the surface. The results utilize the general theory of elliptic surfaces due to Kodaira.

Let E be an elliptic surface having a global section over its base curve X. We assume throughout that the functional invariant $\mathscr{J}$ is nonconstant and that E has no exceptional curves of the first kind in the fibres. We denote by G the homological invariant of E/X. On a Zariski open subset $X_0 \subset X$, G can be viewed as either a locally constant $\boldsymbol{Z} \oplus \boldsymbol{Z}$ sheaf or as a representation $\pi_1(X_0) \to \mathrm{SL}_2(\boldsymbol{Z})$. This representation corresponds to an algebraic vector bundle of rank two on X together with an integrable algebraic connection having regular singular points (Deligne [1], Griffiths [2]), which is known as the Gauss-Manin connection (Katz and Oda [4]). It can be expressed as a second order algebraic differential equation on X having regular singular points. The explicit form of this equation that we shall make use of appears in Stiller [12].

We begin with a brief section of preliminaries, recalling some previous results which relate the geometry of the elliptic surfaces over X to properties of the corresponding differential equations (K-equations, see Stiller [12]).

The first section describes a period mapping from the base curve X to the modular curve M_Γ where $\Gamma \subset \mathrm{SL}_2(\boldsymbol{Z})$ is the global monodromy group of both E/X and the differential equation. Also we give a number of conditions under which $\Gamma = \mathrm{SL}_2(\boldsymbol{Z})$ (see also § 3). When $\Gamma = \mathrm{SL}_2(\boldsymbol{Z})$ the group of $K(X)$-rational division points on the generic fibre (which is an elliptic curve over $K(X)$ the function field of the base curve X) is zero.

In section two we examine a number of invariants of E/X such as the Picard number, the valence of the functional invariant $\mathscr{J}$, the index of the monodromy group Γ in $\mathrm{SL}_2(\boldsymbol{Z})$, and other numerical invariants to determine their behavior when we pass to a generically isogeneous surface over X. The main results are that all of these invariants remain unchanged under generic isogeny! We will utilize the fact that in this case the differential equation does not change (Stiller [12]).