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AN APPLICATION OF GROUPOID COHOMOLOGY

CAROLINE SERIES

We study the structure of analytic measured groupoids as defined by Mackey. It has been observed by Ramsay that an arbitrary groupoid can be thought of as an equivalence relation on its unit space together with a field of isotropy subgroups.

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Introduction. We study the structure of analytic measured groupoids as defined by Mackey [4]. It has been observed by Ramsay [8], Theorem 6.9 that an arbitrary groupoid can be thought of as an equivalence relation on its unit space together with a field of isotropy subgroups. A groupoid homomorphism consists of an orbit preserving mapping of the unit spaces together with a homomorphism of the fields of isotropy subgroups. We formalize this correspondence in the language of group extensions. The discussion is motivated by the observation that if $\mathscr{G}$ is any groupoid we can associate to it $R\mathscr{G}$, the corresponding equivalence relation, and $\Gamma\mathscr{G}$, the field of isotropy subgroups, and there are natural maps $\Gamma\mathscr{G} \to \mathscr{G} \to R\mathscr{G}$. This is a short exact sequence of groupoids, in a sense explained in §1, so that $\mathscr{G}$ may be thought of as an extension of the field $\Gamma\mathscr{G}$ by the equivalence relation $R\mathscr{G}$.

In §2 we construct a cohomology theory for equivalence relations with coefficients in a field of abelian groups, and show that two possible definitions using strict cochains or almost everywhere cochains coincide. In §3 we consider how to reconstruct a groupoid from an equivalence relation and a field of groups. More precisely, an abstract kernel will consist of an equivalence relation and a field of groups together with suitable connecting isomorphisms. Any groupoid gives rise to an abstract kernel and conversely any abstract kernel gives rise to a groupoid provided that a certain obstruction in H^3 vanishes. The methods we use are algebraically an exact analogue of the usual theory of group extensions [5]. It is the author's hope that the language of abstract kernels may prove a more useful viewpoint for the study of groupoids.

Cohomology for groupoids with coefficients in a single abelian group has been discussed by Westman [12], and for the special case