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bo-RESOLUTIONS

MARK MAHOWALD

This paper considers the Adams-Novikov type spectral sequence with bo as the spectrum. The action of the generator of $\pi_8 bo$ in the spectral sequence is completely determined. The result is a complete determination of the v_1-periodic homotopy of the stable sphere.

1. Introduction. Let bo be the Ω-spectrum representing connected real K-theory. This spectrum is a ring spectrum with a unit and $H^*(bo) = A/A(Sq^1, Sq^2)$. (Unless otherwise noted, A is the mod 2 Steenrod algebra, all coefficient groups are $\boldsymbol{Z}_2$, and all spaces are localized at 2.)

Associated to a spectrum with unit, like bo, we have a tower of spectra

$$\begin{array}{ccccccccccc}
S^0 & \longleftarrow & S_1 & \longleftarrow & S_2 & \longleftarrow & \cdots & \longleftarrow & S_s & \longleftarrow & S_{s+1} \longleftarrow \cdots \\
\downarrow & & \downarrow{\scriptstyle id \wedge \iota} & & \downarrow & & & & \downarrow & & \\
bo & & S_1 \wedge bo & & S_2 \wedge bo & & & & S_s \wedge bo & &
\end{array}$$

where $S_s \wedge bo \overset{id \wedge \iota}{\longleftarrow} S_s \leftarrow S_{s+1}$ is a fibration and $l\colon S^0 \to bo$ is the unit. If we use the homotopy functor π^*, we get an exact couple with $E_1^{s\,t} = \pi_{t-s}(S_s \wedge bo)$. Under reasonable hypothesis on the spectrum, $E_\infty^{*,*}$ is an associated graded group of $\pi_*(S^0)$. This is true for bo since $\pi_j(S_s) = 0$ for $j < 3s$ and so for $t - s < 3s$, $E_r^{s,t} = E_\infty^{s,t}$ for large enough r. This spectral sequence will be written $\{E_r(S^0, bo, \pi)\}$.

Clearly $\pi_* bo$ acts on E_1 but d_1 is not a $\pi_* bo$ module map. Nevertheless, if two classes in E_1 are related by the action of a class in $\pi_* bo$ and they both survive to E_∞, then we will say that these two classes are still related in this manner. In particular the class which generates $\pi_8 bo$ is a basic periodicity class which we will call v_1^4. (The name is suggested by BP-theory and is discussed in [3].) A class such that all multiples of it with v_1^{4k} are non zero is called a v_1-periodic class. Classes in E_1 which survive to E_∞ but for which all $\pi_* bo$ compositions except the identity do not survive will be said to generate a $\boldsymbol{Z}_2$ vector space. Our main theorem is:

THEOREM 1.1.

(a) $$\begin{aligned} E_\infty^{0,t}(S^0, bo, \pi) &= \boldsymbol{Z} & t &= 0 \\ &= \boldsymbol{Z}_2 & t &\equiv 1, 2 \bmod 8 \\ &= 0 & &\textit{all other } t\,. \end{aligned}$$