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PARTITIONS OF GROUPS AND COMPLETE MAPPINGS

RICHARD J. FRIEDLANDER, BASIL GORDON
AND PETER TANNENBAUM

Let G be an abelian group of order n and let k be a divisor of $n-1$. We wish to determine whether there exists a complete mapping of G which fixes the identity element and permutes the remaining elements as a product of disjoint k-cycles. We conjecture that if G has trivial or noncyclic Sylow 2-subgroup then such a mapping exists for every divisor k of $n-1$. Several special cases of the conjecture are proved in this paper. We also prove that a necessary condition for the existence of such a map holds for every k when G is cyclic.

1. **Introduction.** A complete mapping of a group G is defined to be a bijection $\phi: G \rightarrow G$ such that the mapping $\theta: g \rightarrow g^{-1}\phi(g)$ is also bijective. (Some authors refer to θ, rather than ϕ, as the complete mapping.) If the permutation $\begin{pmatrix} b_1 \, b_2 \cdots b_n \\ c_1 \, c_2 \cdots c_n \end{pmatrix}$ is a complete mapping of G and $g \in G$, then $\begin{pmatrix} b_1 & b_2 & \cdots & b_n \\ c_1 g & c_2 g & \cdots & c_n g \end{pmatrix}$ is clearly also a complete mapping of G. By suitable choice of g, we can therefore suppose that $b_n = c_n = 1$. Then the complete mapping can be viewed as a permutation $\begin{pmatrix} b_1 \, b_2 \cdots b_{n-1} \\ c_1 \, c_2 \cdots c_{n-1} \end{pmatrix}$ of the nonidentity elements of G. The permutation $\begin{pmatrix} b_1 \, b_2 \cdots b_{n-1} \\ c_1 \, c_2 \cdots c_{n-1} \end{pmatrix}$ is cyclic if and only if it can be written in the form $\begin{pmatrix} a_1 \, a_2 \cdots a_{n-1} \\ a_2 \, a_3 \cdots a_1 \end{pmatrix}$, where $a_1^{-1}a_2, a_2^{-1}a_3, \cdots, a_{n-1}^{-1}a_1$ are all distinct. In this case we say that G is an *R-sequenceable* group with R-sequencing $a_1, a_2, \cdots, a_{n-1}$. Thus a group G is R-sequenceable if and only if it has a complete mapping which fixes the identity element and permutes the remaining elements cyclically. In [2], we determined several infinite classes of R-sequenceable abelian groups (see (1)-(6) below).

In this paper, we generalize the notion of R-sequenceability by asking which groups G of order n have the property that, given any regular partition $k + k + \cdots + k$ (d terms) of $n-1$, there exists a complete mapping of G which fixes the identity element and permutes the remaining elements as a product of d disjoint k-cycles. We call such a mapping a *k-regular complete mapping of G*. That is, given any divisor k of $n-1$, a k-regular complete mapping of G is a permutation $\begin{pmatrix} b_1 \, b_2 \cdots b_{n-1} \\ c_1 \, c_2 \cdots c_{n-1} \end{pmatrix}$ of the nonidentity elements of G whose