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ON THE STRUCTURE OF HYPER-REAL z-ULTRAFILTERS

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This paper investigates the structure of hyper-real z-ultrafilters on completely regular, Hausdorff spaces in an attempt to describe their structure in manageable terms. A consequence of this investigation is a scheme for classifying these z-filters based on the complexity of their structure. It is shown that the real numbers with the usual topology exhibit hyper-real z-ultrafilters within each category of the classification. The paper closes with a discussion of how the action of z-filters in one category influence those in the other categories with particular applications to the study of $C^{\#}(X)$.

The study of βX, the Stone-Čech compactification of the completely regular Hausdorff space X, has proven to be a useful yet complex subject. Many results in the area (for example, in the non-homogeneity question of $\beta X \backslash X$) have had their foundations in an understanding of the structure of the z-ultrafilters on X. For example, remote points, although defined as points in $\beta X \backslash \nu X$ which are not in the βX-closure of a discrete subspace of X, were first shown to exist by showing the existence of a free z-ultrafilter no element of which is nowhere dense.

This paper investigates the structure of hyper-real z-ultrafilters on completely regular, Hausdorff spaces. The object is to describe these filters (or, more accurately, their bases) in terms that provide an insight into their structure. This is done by observing that every hyper-real z-ultrafilter has the structure of a free ultrafilter on $\boldsymbol{N}$, the discrete space of natural numbers, underlying it. This underlying structure is formalized in terms of a skeleton. It is shown that many hyper-real z-ultrafilters can be realized as being built up from a skeleton using other z-ultrafilters (one for each element in $\boldsymbol{N}$) in a way which closely resembles that of a subdirect product. In such cases, the original z-ultrafilter is said to be real-decomposable or hyper-decomposable depending on whether or not these other z-ultrafilters can be chosen to be real z-nltrafilters. Although there are hyper-real z-ultrafilters that defy such decomposition, the present theory provides insight into the structure of a significant set of hyper-real z-ultrafilters as shown by the fact that the set of points associated with the decomposable z-ultrafilters is dense in $\beta X \backslash \nu X$. A consequence of the present theory is a classification of the hyper-real z-ultrafilters into three classes; namely, real-decomposable, hyper-