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REPRESENTATION OF COMPACT AND WEAKLY COMPACT OPERATORS ON THE SPACE OF BOCHNER INTEGRABLE FUNCTIONS

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If X^* has the Radon-Nikodym property, then for every compact operator $T: L_1(\mu, X) \to Y$ there is a bounded function $g: \Omega \to L(X, Y)$ that is measurable for the uniform operator topology on $L(X, Y)$ such that

$$T(f) = \int_{\Omega} fg d\mu$$

for all f in $L_1(\mu, X)$. The same result holds for weakly compact operators if X^* is separable Schur space. These representations yield Radon-Nikodym theorems for operator valued measures and a generalization of a theorem of D. R. Lewis.

The representation of linear operators on the Banach space $L_1(\mu, X)$ of Bochner integrable functions, has been the object of much study for the past forty years. Dunford and Pettis began this investigation in 1940 [6] with the representation of weakly compact and norm compact operators on $L_1(\mu)$ by a Bochner integral. Their work was based on an earlier paper of Pettis [9] and was complemented by the work of Phillips [11]. More recently, the theory of liftings has been used by Dinculeanu [5] and others to obtain a representation for the general linear operator on $L_1(\mu, X)$. In this paper we will use methods in the spirit of Dunford, Pettis, and Phillips to show that if X^* has the Radon-Nikodym property, then the compact operators on $L_1(\mu, X)$ are representable by measurable kernels and if X^* is a separable Schur space (i.e., weakly convergent sequences converge in norm) then the weakly compact operators on $L_1(\mu, X)$ are representable by measurable kernels. As corollaries, we obtain a Radon-Nikodym theorem for operator-valued measures and a generalization of a theorem of D. R. Lewis [4, p. 88] on weakly measurable functions that are equivalent to norm measurable functions.

Throughout this paper (Ω, Σ, μ) is a finite measure space and X, Y and Z are Banach spaces with duals X^*, Y^*, and Z^* respectively. The space of all bounded linear operators from X to Y will be denoted by $L(X, Y)$. The subspaces of $L(X, Y)$ consisting of all the weakly compact and norm compact operators from X to Y will be denoted by $W(X, Y)$ and $K(X, Y)$. The space $L_1(\mu, X)$ is the space of μ-Bochner integrable functions on Ω with values in X and