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THE ASYMMETRIC PRODUCT OF THREE HOMOGENEOUS LINEAR FORMS

A. C. WOODS

Let $L_i = \sum_{j=1}^{3} a_{ij}x_j$, $i = 1, 2, 3$, be three linear forms in the variables x_1, x_2, x_3 with real coefficients a_{ij}. A theorem of Davenport asserts that, if $|\det(a_{ij})| = 7$, then there exist integers u_1, u_2, u_3, not all zero, such that

$$\left| \prod_{i=1}^{3} L_i(u_1, u_2, u_3) \right| \leqq 1 \, .$$

Under the same hypothesis, W. H. Adams has asked whether, given a positive real number u, there exist integers u_1, u_2, u_3, not all zero, such that

$$-u^{-1} \leqq L_1(u_1, u_2, u_3)L_2(u_1, u_2, u_3)\,|L_3(u_1, u_2, u_3)| \leqq u \, .$$

Our objective is to prove this conjecture.

Davenport gave several proofs of his theorem [3], and other proofs have been given by Chalk and Rogers [2] and Mordell [8]. Isolation results, notably those of Davenport [6] and Swinnerton-Dyer [10], show that Adams conjecture is true for real u in some open interval containing 1.

The set of points (L_1, L_2, L_3) in R_3, formed as the variables range over all integral values, is a lattice Λ of determinant $d(\Lambda) = |\det(a_{ij})|$. In terms of Λ, our result is as follows.

THEOREM. *If $d(\Lambda) = 7$, then there exists a point (x_1, x_2, x_3) of Λ, other than the origin, such that*

$$-u^{-1} \leqq x_1x_2|x_3| \leqq u \, ,$$

with the equality sign being necessary only if $u = 1$.

The method of proof is the projective one due to Davenport [3]. We begin with three lemmas.

LEMMA 1. *If x, y, z, t are real nuumbers with $1 < t^2 \leqq 1.9$, such that the inequality*

$$(1) \qquad -t^2 < (n + x)(n + y)|n + z| < 1$$

is not solvable in integers n, then

$$(2) \qquad \varphi = (x - y)^2 + (y - z)^2 + (z - x)^2 > 14t \, .$$

We note that this is a generalization of a lemma due to