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SOLUBILITY OF FINITE GROUPS ADMITTING A FIXED-POINT-FREE AUTOMORPHISM OF ORDER *rst* I

PETER ROWLEY

The 'fixed-point-free automorphism conjecture' asserts that if a finite group G admits a fixed-point-free automorphism group A (and, if A is noncyclic, further suppose that $(|G|, |A|) = 1$), then G is soluble. This paper is the first in a four part series, which considers the above conjecture when A is cyclic of order rst where r, s and t are distinct prime numbers.

1. Introduction. Suppose G is a finite group. For A a subgroup of the automorphism group of G we say that A acts fixed-point-freely upon G if and only if $C_G(A) = \{g \in G \mid a(g) = g, \forall a \in A\} = \{1\}$. When $A = \langle \alpha \rangle$ is cyclic we sometimes say α acts fixed-point-freely upon G.

Let r, s and t denote distinct prime numbers. The main result to be proved here is

THEOREM 1.1. *A finite group which admits a coprime fixed-point-free automorphism of order rst is soluble.*

In [15] the above result is obtained with the additional assumption that rst is a non-Fermat number (for the definition of a non-Fermat number see § 4). The main result of [15] has been further extended in [17] where the 'fixed-point-free automorphism conjecture' is established for automorphisms whose order is a non-Fermat square-free number. The 'fixed-point-free automorphism conjecture' asserts the following.

If a finite group G admits a fixed-point-free automorphism group A (and, if A is noncyclic, further suppose that $(|G|, |A|) = 1$), then G is soluble.

References for other works which contribute to the solution of this problem may be found in [13] and [16].

We now review the strategy of the proof of Theorem 1.1. A substantial part of our arguments will be in the context of a minimal situation. So let the pair $(G, \langle \alpha \rangle)$ be a counterexample to Theorem 1.1 chosen so that $|G| + |\langle \alpha \rangle|$ is minimal. Lemma 3.13 demonstrates, in such a group, the existence of certain α-invariant nilpotent Hall subgroups. Let L and M denote (respectively) α-invariant nilpotent Hall λ- and μ-subgroups of G. By (2.22) the number of maximal