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BASE CHANGE FOR TEMPERED IRREDUCIBLE REPRESENTATIONS OF GL $(n, \boldsymbol{R})$

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Let π be a tempered irreducible representation of GL $(n, \boldsymbol{R})$. We prove the expected relation between the characters of π and its base change lifting.

0. Introduction. To each irreducible representation π of GL $(n, \boldsymbol{R})$ is associated its "base change lifting", an irreducible representation Π of GL $(n, \boldsymbol{C})$. It is expected that the characters of these two representations are related in a certain way, at least if π is tempered, and this relation has in fact been proved for GL $(2, \boldsymbol{R})$ by Shintani [4], and for representations of GL $(n, \boldsymbol{R})$ induced from unramified quasicharacters of a minimal parabolic subgroup by Clozel [1]. The purpose of this paper is to prove the relation for arbitrary tempered irreducible representations of GL $(n, \boldsymbol{R})$.

The proof involves computations not unlike those used to calculate the character of an induced representation. The representations in question are all induced from parabolic subgroups whose Levi components are products of copies of GL (2) and GL (1), so we are able to use Shintani's results for GL (2) as a starting point. It is to be expected that a similar "inductive step" can be proved for the general quasi-split connected real reductive group, but technical problems make that more difficult.

1. Notation and preliminaries. Let $G = \mathrm{GL}(n)$, $n \geqq 3$. Every irreducible tempered representation π of G_R is induced from a cuspidal parabolic subgroup P_R. After conjugation, we may assume $P = MN$, where the Levi component M consists of 2×2 and/or 1×1 blocks along the diagonal and N, the unipotent radical of P, consists of upper triangular matrices with diagonal entries all equal to 1 and with zero for those entries which lie inside the blocks of M. Thus $M \cong \mathrm{GL}(2)^k \times \mathrm{GL}(1)^{n-2k}$. Also let $K = U(n)$.

We recall some remarks about σ-conjugacy (see, e.g., [1], § 2). Write g^σ for the complex conjugate of an element $g \in G_C$. Two elements $g, g' \in G_C$ are σ-conjugate if $g = h^\sigma g' h^{-1}$, for some $h \in G_C$. If $g \in G_C$, its norm is defined by $Ng = g^\sigma g$. If g and g' are σ-conjugate, then Ng and Ng' are conjugate in G_C. As usual, we write G'_C for the regular elements of G_C; we shall say g is σ-regular if $Ng \in G'_C$, and write G''_C for the σ-regular elements of G_C. The complement of G''_C is a real analytic subvariety of measure zero.