PACIFIC JOURNAL OF MATHEMATICS
Vol. 93, No. 1, 1981

EMBEDDINGS OF THE PSEUDO-ARC IN E^2

WAYNE LEWIS

In this paper, we show that there exists an embedding, P_s, of the pseudo-arc in the plane such that any two accessible points lie in distinct composants of P_s. We also show that there are $c=2^{\omega_0}$ distinct embeddings of the pseudo-arc in the plane, including for each positive integer n, one with exactly n composants accessible. This answers some questions and a conjecture of Brechner.

For definitions and notation of chain (from p to q), link, crooked, etc., see [1] and [7]. The links of our chains will always be the interiors of disks, and if two links of a chain intersect their intersection is the interior of a disk. When a chain D refines a chain C, we shall always require that the closure of each link of D be contained in a link of C.

First we describe the special embedding P_s, then prove Brechner's conjecture that any two distinct accessible points of P_s lie in distinct composants. Let C_0 be a chain in E^2 from point p to point q which runs straight across from left to right horizontally. Let C_1 be a chain also running from p to q which is crooked in C_0 and descending, as in Figure 1. If we think of C_1 as straightened out

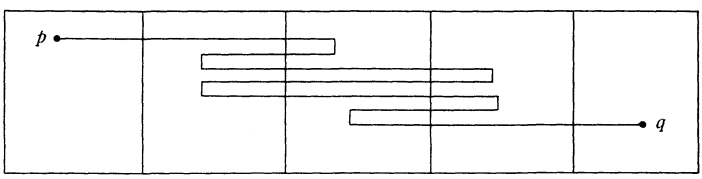

FIGURE 1
Only the nerve of C_1 in C_0 is shown.

with p on the left and q on the right, then C_2 is a chain from p to q which is crooked in C_1 and ascending. We continue in this manner, alternating descending and ascending chains, so that C_i runs from p to q, mesh $(C_i) < 1/2^i$, C_{i+1} refines and is crooked in C_i, and C_{i+1} is descending (ascending) in C_i if i is even (odd). The pseudo-arc P_s is $\bigcap_{i\in\omega_0} C_i^*$. (If A is a collection of sets, A^* is the union of A.)

THEOREM. *Any two distinct accessible points of P_s are in distinct components.*

Proof. We can draw horizontal rays to the left from p and to