PACIFIC JOURNAL OF MATHEMATICS
Vol. 93, No. 1, 1981

SUPERCUSPIDAL COMPONENTS OF THE QUATERNION WEIL REPRESENTATION OF $SL_2(\mathfrak{k})$

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Let $\mathfrak{k}$ be a p-adic field of odd residual characteristic. It is known that all but one summand of the quaternion Weil representation are supercuspidal. These summands are precisely identified in terms of corresponding summands of quadratic extension Weil representations.

1. Let $\mathfrak{k}$ be a p-adic field of odd residual characteristic. From [6] we know that all supercuspidal representations of $G = SL_2(\mathfrak{k})$ occur as summands of various Weil representations associated with quadratic extensions of $\mathfrak{k}$. It is also known that the Weil representation associated to the unique quaternion division algebra over $\mathfrak{k}$ decomposes into a direct sum of irreducible representations, all but one of which are supercuspidal. The object of this paper is to show just how these representations correspond to summands of quadratic extension Weil representations. The methods depend heavily on [3]. The primary motivation for this paper was the problem of decomposing tensor products of certain supercuspidal representations of G. The authors have been told that some similar computations have been worked out by J. Shalika and W. Casselman.

2. In this paper, the ring of integers in $\mathfrak{k}$ and its prime ideal are denoted respectively by $\mathfrak{o}$ and $\mathfrak{p}$. We choose a generator π of $\mathfrak{p}$ and a non square unit ε in $\mathfrak{o}$. The order of the residue class field will be denoted by q.

For $\theta \in \{\pi, \varepsilon, \varepsilon\pi\}$, we let $\mathfrak{o}_\theta$ and $\mathfrak{p}_\theta$ denote the ring of integers in $\mathfrak{k}(\sqrt{\theta})$ and its prime ideal respectively. Trace and norm of $\mathfrak{k}(\sqrt{\theta})$ over $\mathfrak{k}$ are written τ_θ and ν_θ respectively.

The quaternion division algebra over $\mathfrak{k}$ will be denoted by D. Its integers will be denoted by A and the prime ideal of A will be P. The reduced norm and trace of D over $\mathfrak{k}$ are written respectively ν_D and τ_D. The set $\{1, i, j, k\}$ is a basis of D over $\mathfrak{k}$ where $i^2 = \varepsilon$, $j^2 = \pi$, and $ij = -ji = k$. There are convenient imbeddings of $\mathfrak{k}(\sqrt{\varepsilon})$ and $\mathfrak{k}(\sqrt{\pi})$ in D where $\mathfrak{k}(\sqrt{\varepsilon}) = \{a + bi: a, b \in \mathfrak{k}\}$ and $\mathfrak{k}(\sqrt{\pi}) = \{a + bj: a, b \in \mathfrak{k}\}$. Let S be a complete set of residues of $\mathfrak{o}_\varepsilon/\mathfrak{p}_\varepsilon$ (and thus of A/P) consisting of zero and roots of unity. Then for any $z \in D$ we may write $z = \sum_{n=N}^{\infty} \alpha_n j^n$ where each $\alpha_n \in S$.

Since π can be chosen to be any element in $\mathfrak{k}$ that generates $\mathfrak{p}$, we will generally consider only the cases $\theta = \varepsilon$ and $\theta = \pi$.

3. From [3], we recall some information on the representations