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MANIFESTLY DYNAMIC FORMS IN THE CARTAN-HAMILTON TREATMENT OF CLASSICAL FIELDS

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Our intent is to show that certain differential forms, which are manifestly closed on the motions of a classical field system, are Hamiltonic in the sense of generating a canonical vector field, or are equivalent to Hamiltonic forms.

1. Introduction. Let K be a differentiable manifold of dimension at least m and let α be a differential form of degree m defined in K. Familiar considerations from the calculus of variations then leads to certain m-dimensional submanifolds of K, the extremals for α[1].

For $m = 1$ this reduces to the Hamiltonian formalism of dynamics, and in this case a dynamic variable is a function on K which is constant on the extremals. The generalization to $m \geqq 1$ is to define an $(m-1)$-form φ to be a *dynamic* form for α if the restriction $d\varphi|E$ of its differential to each extremal E vanishes [1, 2, 3, 4, 5, 6, 7, 8].

A special class of dynamic forms are the *Hamiltonic* forms [4] [7, 111]. An $(m-1)$-form φ in K is Hamiltonic if there is a vector field U on K such that $d\varphi$ coincides with the interior product $U \rfloor d\alpha$. For φ Hamiltonic as above and ψ any $(m-1)$-form, a new $(m-1)$-form

$$\{\varphi, \psi\} = U \rfloor d\psi$$

has been defined in [4], and called the *Poisson* bracket.

In the theory of Kijowski, the dynamic forms whose support has a certain compactness property [6, 112] are used to define observables by being integrated over extremals. Two dynamic forms whose difference vanishes on every extremal are called equivalent.

Our intent is to give a sufficient condition that a form be equivalent to a Hamiltonic form. The condition ("manifestly dynamic form") does not involve the postulation of a canonic vector field U. The precise definitions require a preliminary discussion of the type of m-form α to be considered.

The m-form α is required to originate in a Lagrangian [4]. The α is supposed to be defined on a first-order jet bundle

$$J^{(1)}(Q, \boldsymbol{R}^m)$$