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REDUCING THE ORDER OF A LAGRANGIAN

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Consider a Lagrangian density L defined on R^m for a system with configuration space Q. Let the order of the highest order derivatives in L be N. Then the Euler equations are generally of order $2N$. We present ways of replacing L by other Lagrangian densities L' on R^m which are of order 1 and in fact linear in the derivatives. This is done by introducing roughly n times

$$\binom{m+N-1}{N-1}+\binom{m+N}{N}-2$$

additional variables, where n is the dimension of Q.

One of these L' (denoted by $L^\wedge$) is such that its Euler equations have a canonical form reducing to that of Hamilton for $N=m=1$.

2. **Notation.** A Lagrangian density L is a function F of several variables, first the cartesian coordinates

$$t^1, \cdots, t^m$$

in $\boldsymbol{R}^m$, and then the coordinates

$$x^1, \cdots, x^n$$

in some manifold Q, and also

$$x_i^\lambda\,, \quad x_{ij}^\lambda\,, \quad x_{ijk}^\lambda\,, \cdots \tag{2.1}$$

where roman indices range from 1 to m and greek indices from 1 to n. Here $i \leqq j$, $i \leqq j \leqq k$. The greatest number of subscripts on any letter occurring in L is the *order* of L. L is used to define a functional Λ on the class of suitable differentiable maps f from $\boldsymbol{R}^m$ to Q. In $L = F(t, x, \cdots)$ one replaces each x^λ by the component $f^\lambda = x^\lambda \circ f$, and replaces x_i^λ by $\partial f^\lambda/\partial t^i$, x_{ij}^λ by $\partial^2 f^\lambda/\partial t^i \partial t^j$ ect., and defines $\Lambda(f)$ by

$$\int_I L \circ Df dt^1 \cdots dt^m\,.$$

The notation $L \circ Df$ is supposed to show that the values of f and its derivatives have been inserted into L.

We are going to meet variables with suffixes of their own, and indeed varying from 1 to m, so this notation will have to be amended. If y_{14} is some variable occurring in L, let $w = y_{14}$, then one