PACIFIC JOURNAL OF MATHEMATICS
Vol. 93, No. 2, 1981

THE SHEAF OF H^p-FUNCTIONS IN PRODUCT DOMAINS

SERGIO E. ZARANTONELLO

Let $W = W_1 \times W_2 \times \cdots \times W_n$ be a bounded polydomain in C^n such that the boundary of each W_i consists of finitely many disjoint Jordan curves. The correspondence that assigns to every relatively open polydomain V in $\bar{W}$ (the closure of W) the Hardy space $\mathscr{H}^p(V \cap W)$, defines a sheaf $\hat{\mathscr{H}}^p_W$ over $\bar{W}$. This sheaf is locally determined in the sense that $\Gamma(\bar{W}, \hat{\mathscr{H}}^p_W)$ is canonically isomorphic to $\mathscr{H}^p(W)$. In this paper we prove, for any $0 < p < \infty$ and all integers $q \geq 1$, that the cohomology groups $H^q(\bar{W}, \hat{\mathscr{H}}^p_W)$ are trivial.

I. Introduction. The *Hardy spaces* $\mathscr{H}^p(U^n)$, $0 < p < \infty$, for the unit polydisc U^n, consist of all functions F which are holomorphic in U^n and satisfy

$$\sup_{0<r<1} \int_0^{2\pi} \cdots \int_0^{2\pi} |F(re^{i\theta_1}, \cdots, re^{i\theta_n})|^p d\theta_1 \cdots d\theta_n < +\infty .$$

The observation ([9, Exercise 3.4.4(b), p. 52]) that $F \in \mathscr{H}^p(U^n)$ if and only if F is holomorphic and $|F|^p$ has an n-harmonic majorant in U^n, leads to a definition of Hardy spaces for arbitrary product domains; the requirement now being that F be holomorphic and $|F|^p$ have an n-harmonic majorant in the polydomain in question.

The symbol $\mathscr{H}^p$ can thus be regarded as a presheaf on the polydomains in C^n. In this paper we concern ourselves with the sheaf induced by $\mathscr{H}^p$ on the closure of a polydomain, and prove, under certain topological restrictions, that the corresponding cohomology groups are trivial.

Specifically, let $W = W_1 \times W_2 \times \cdots \times W_n$ be a bounded polydomain in C^n, and suppose each W_i is bounded by finitely many disjoint Jordan curves. The correspondence that assigns to each relatively open product domain V in $\bar{W}$ (the closure of W) the linear space $\mathscr{H}^p(V \cap W)$, defines a sheaf $\hat{\mathscr{H}}^p_W$ over $\bar{W}$. This sheaf is *locally determined*, i.e., $\Gamma(\bar{W}, \hat{\mathscr{H}}^p_W)$ is canonically isomorphic to $\mathscr{H}^p(W)$. Our goal is to prove, for any such W, for $0 < p < \infty$, and for all integers $q \geqq 1$, that the cohomology groups $H^q(\bar{W}, \hat{\mathscr{H}}^p_W)$ are trivial.

In [8] A. Nagel proved similar results for a wide class of sheaves of holomorphic functions satisfying boundary conditions in polydomains. Although Nagel's methods can be applied to the sheaves $\hat{\mathscr{H}}^p_W$ when $1 < p < \infty$, the cases $0 < p \leqq 1$ present difficulties. Instead, as in the earlier papers [12], [13], we follow the approach