PACIFIC JOURNAL OF MATHEMATICS
Vol. 93, No. 2, 1981

INVARIANT SUBSPACES FOR FINITE MAXIMAL SUBDIAGONAL ALGEBRAS

KICHI-SUKE SAITO

Let M be a von Neumann algebra with a faithful, normal, tracial state τ and H^∞ a finite, maximal, subdiagonal algebra in M. If $1 \leqq p < s \leqq \infty$, then there is a one-to-one correspondence between left-(resp. right-) invariant subspaces of the noncommutative Lebesgue space $L^p(M, \tau)$ and those of $L^s(M, \tau)$.

1. Introduction. Let M be a von Neumann algebra with a faithful, normal, tracial state τ and let H^∞ be a finite, maximal, subdiagonal algebra in M. A number of authors have investigated the structure of the invariant subspaces for H^∞ acting on the noncommutative Lebesgue space $L^p(M, \tau)$ (cf. [3], [4], [5] and [6]). In [6], we showed that, if $\mathfrak{M}$ is a left-(resp. right-) invariant subspace of $L^p(M, \tau)$, $1 \leqq p < \infty$, then $\mathfrak{M}$ is the closure of the space of bounded elements it contains.

In this paper, we shall show that, if $1 \leqq p < s \leqq \infty$, then there is a one-to-one correspondence between left- (resp. right-) invariant subspaces $\mathfrak{M}_p$ of $L^p(M, \tau)$ and left- (resp. right-) invariant subspaces $\mathfrak{M}_s$ of $L^s(M, \tau)$, such that $\mathfrak{M}_s = \mathfrak{M}_p \cap L^s(M, \tau)$ and $\mathfrak{M}_p$ is the closure in $L^p(M, \tau)$ of $\mathfrak{M}_s$. This is of course true in the weak*-Dirichlet algebras setting (cf. [2, p. 131]) and this is attractive to study the structure of the invariant subspaces of $L^p(M, \tau)$.

2. Let M be a von Neumann algebra with a faithful, normal, tracial state τ. We shall denote the noncommutative Lebesgue spaces associated with M and τ by $L^p(M, \tau)$, $1 \leqq p < \infty$ (cf. [7]). As is customary, M will be identified with $L^\infty(M, \tau)$. The closure of a subset S of $L^p(M, \tau)$ in the L^p-norm will be denoted by $[S]_p$; $[S]_\infty$ will denote the closure of S in the σ-weak topology on $L^\infty(M, \tau)$.

DEFINITION 1. Let H^∞ be a σ-weakly closed subalgebra of M containing the identity operator 1 and let Φ be a faithful, normal expectation from M onto $D = H^\infty \cap H^{\infty *} (H^{\infty *} = \{x^* : x \in H^\infty\})$. Then H^∞ is called a finite, maximal, subdiagonal algebra in M with respect to Φ and τ in case the following conditions are satisfied: (1) $H^\infty + H^{\infty *}$ is σ-weakly dense in M; (2) $\Phi(xy) = \Phi(x)\Phi(y)$, for all $x, y \in H^\infty$; (3) H^∞ is maximal among those subalgebras of M satisfying (1) and (2); and (4) $\tau \circ \Phi = \tau$.