PACIFIC JOURNAL OF MATHEMATICS
Vol. 93, No. 2, 1981

A UNIQUENESS THEOREM FOR NAVIER-STOKES EQUATIONS

CHUN-MING MA

In this paper we consider the initial boundary value problem for the Navier-Stokes equations in several types of unbounded three-dimensional domains Ω. We prove uniqueness within a class of solutions, which we call "weak class H_0 solutions", whose members satisfy the integrability conditions ∇u, $\Delta u \in L^2(0, T; L_2(\Omega))$. Moreover, the solutions are shown to depend continuously on their initial values. The results are based, primarily on establishing a simple characterization of a certain space $H_0(\Omega)$ of solenoidal functions.

For exterior domains, we have already given such a characterization of the space $H_0(\Omega)$ in Ma [14]. However, the proof given here is simpler and more direct and yields the result for "aperture domains" as well (i.e., for domains considered by Heywood [8] in studying flow through a hole in a wall).

Our uniqueness theorem should be compared with one given recently by Heywood. In [9], Heywood used our original characterization of the space H_0 to prove uniqueness in exterior domains for solutions satisfying the integrability conditions ∇u, $\Delta u \in L^2(0, T; L^2(\Omega))$ and $\nabla u_t \in L^2(\varepsilon, T; L^2(\Omega))$, for all positive $\varepsilon < T$. Here, we are able to drop the integrability condition for ∇u_t by using a technique introduced in the context of "finite energy" solutions by Prodi [15]; see also Serrin [17]. The main advantage in giving the uniqueness theorem as we do here, without Heywood's integrability condition for ∇u_t, is that one can then consider a larger class of forces. If one considers arbitrary forces, with $\nabla f \in L^2(0, T; L^2(\Omega))$, the integrability condition for ∇u_t is not known, and quite possibly does not hold; see [10]. However, for such forces, generalized solutions satisfying the conditions of our uniqueness theorem do exist. This is proved in the concluding section of the present paper.

Our results should also be compared with a remarkable new uniqueness theorem of Fabrizio [3], which appeared as we were finishing this work. This theorem (Theorem 1 in [3]) requires even less than ours in the way of integrability conditions; it is merely required that the difference of two solutions should belong to $L^s(\Omega \times (0, T))$, for some $s > 1$. On the other hand, it is apparently given only for an exterior domain (though this is not really made clear) and does not provide the continuous dependence of solutions on their initial values. Further, Fabrizio's theorem is based on several