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NOTES ON THE FEYNMAN INTEGRAL, I

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We extend somewhat and simplify substantially some of the recent work of Cameron and Storvick involving the analytic Feynman integral of certain functions on Wiener space of the form $F(x)=\exp\left\{\int_a^b \theta(t,\ x(t))dt\right\}$; here θ is a complex-valued function on $(a,\ b]\times R$ and x is an element of Wiener space, that is, a continuous function on $[a,\ b]$ which vanishes at a.

1. Introduction. In a recent paper [2], Cameron and Storvick treat a Banach algebra S of functions on Wiener space which are a kind of stochastic Fourier transform of Borel measures on $L_2[a,\ b]$. (Precise definitions will be given in the next section.) For such functions they show that the analytic Feynman integral, defined by analytic continuation of the Wiener integral, exists, and they give a formula for this Feynman integral. The work in [2] is related to Albeverio and Høegh-Krohn's beautiful theory [1] of infinite dimensional oscillatory integrals ("Fresnel integrals") as well as to [5]. Cameron and Storvick's work is highly promising and has some appealing features. For example, as we will show in a later note, the existence of the Feynman integral for certain qudratic potentials can be established without having to construct special spaces, quadratic forms, etc. to fit the particular problem of interest.

The main purpose of this note is to show that a crucial part of [2] can be substantially simplified. Let $\boldsymbol{R}$, $\boldsymbol{C}$ denote the real and complex numbers respectively. Let θ map $(a, b]\times \boldsymbol{R}$ to $\boldsymbol{C}$. Let $C[a, b]$ denote Wiener space; that is, the space of $\boldsymbol{R}$-valued continuous functions on $[a, b]$ which vanish at a. Let m denote Wiener measure on $C[a, b]$. Under certain hypotheses on θ, Cameron and Storvick show that the function

$$(1.1)\qquad F(x) = \exp\left\{\int_a^b \theta(t,\ x(t))dt\right\},\ x \text{ in } C[a,\ b]\ ,$$

belongs to the Banach algebra S and hence possesses an analytic Feynman integral. This result depends on some rather elaborate machinery; for example, their spaces $\mathscr{M}'$, S', $\mathscr{M}''$, S'', $\mathscr{M}_n''$, S_n'', $S_n^\wedge$, $\mathscr{M}_n^\wedge$ are all part of this picture. We give a simpler proof of this result avoiding the machinery. We also extend their result somewhat, but it is the simplification that is the main point. It should be mentioned that the results on $\mathscr{M}'$, S', $\mathscr{M}''$, etc. are