PACIFIC JOURNAL OF MATHEMATICS
Vol. 93, No. 2, 1981

# DETECTING THE DISJOINT DISKS PROPERTY

ROBERT J. DAVERMAN

**This paper explores conditions under which a metric space $S$ satisfies the following Disjoint Disks Property: any two maps of the standard 2-cell $B^2$ into $S$ can be approximated by maps having disjoint images. Among its many applications, it provides a proof that if $Y$ is the cell-like image of an $n$-manifold ($n \geq 3$), then $Y \times E^2$ has the Disjoint Disks Property, which implies that $Y \times E^2$ is a manifold. It adds further evidence for the unifying force of this property by giving comparatively easy proofs for established facts about certain decomposition spaces that are manifolds.**

The significance of the Disjoint Disks Property is made manifest by its role in a recently-proved fundamental result about cell-like decompositions, due to R. D. Edwards [17]: if an ANR $X$ is the proper cell-like image of an $n$-manifold $M$ ($n \geqq 5$) and satisfies the Disjoint Disks Property, then $X$ is an $n$-manifold homeomorphic to $M$.

J. W. Cannon, who obtained a fairly strong partial result of this type [12], receives the credit for focusing attention on the Disjoint Disks Property and making plausible the claim that it should be the crucial additional feature forcing such an ANR to be a manifold [10], [11], [12]. Like so much of the subject of manifold decompositions, origins of this property can be traced to early work of R. H. Bing, in this case where he developed methods for determining whether certain cellular decompositions of Euclidean 3-space $E^3$ were shrinkable [5], [6].

Enhancing the significance of the Disjoint Disks Property is another fundamental result, announced by F. Quinn [28]: if $X$ is a generalized $n$-manifold (namely, a finite dimensional ANR such that for all $x \in X$ $H_*(X, X - \{x\})$ coincides with $H_*(E^n, E^n - \{0\})$ and $n \geqq 5$, then there exists a cell-like proper map of an $n$-manifold $M$ onto $X$. The combination of Edwards' and Quinn's work presents a basic characterization of topological manifolds: for $n \geqq 5$ a space $X$ is an $n$-manifold iff it is a generalized $n$-manifold that satisfies the Disjoint Disks Property.

This paper aims toward applications of this characterization, primarily within decomposition theory itself. At the heart of most applications here is the well-known result [31] (see also [1] or the discussion in [10, p. 323]) that if an ANR $X$ is the proper cell-like image of an $n$-manifold $M$ (without boundary), then $X$ is a gener-