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DIFFERENTIABLY k-NORMAL ANALYTIC SPACES AND EXTENSIONS OF HOLOMORPHIC DIFFERENTIAL FORMS

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In this paper the concept of normality for a complex analytic space X is strengthened to the requirement that every local holomorphic p-form, for all $0 \leq p \leq$ some integer k, defined on the regular points of X extend across the singular variety. A condition for when this occurs is given in terms of a notion of independence, in the exterior algebra $\Omega^*_{\Delta^N}$, of the differentials $dF_1, \cdots, dF_r$ of local generating functions F_i of the ideal of X in some ambient polydisc $\Delta^N \subset C^N$. One result is that for a complete intersection, "k-independent implies $(k-2)$-normal" (precise definitions are given below), which extends some ideas of Oka, Abhyankar, Thimm, and Markoe on criteria for normality.

Recall that a complex space $(X, \mathcal{O}_X)$ is *normal* at a point $x \in X$ if every bounded holomorphic function defined on the regular points in a punctured neighborhood of x extends analytically to the full neighborhood. This is equivalent to the condition that the ring $\mathcal{O}_{X,x}$ be integrally closed in its field of quotients, and except for regular points x in dimension 1 the boundedness requirement is irrelevant: if $\dim X > 1$, $x \in X$ is normal $\Leftrightarrow$ for all sufficiently small neighborhoods U of x the restriction of sections $\Gamma(U, \mathcal{O}_X) \to \Gamma(U - \sum, \mathcal{O}_X)$ is an isomorphism, for $\sum$ the set of singular points of X. In 1974 A. Markoe [6] observed that the basic modern ideas in the topic of cohomology with supports gives a very simple criterion of normality in terms of the homological codimension of the structure sheaf:

THEOREM (*Markoe*). *Let $(X, \mathcal{O}_X)$ be a reduced complex space with singular set $\sum$. Then $\forall x \in X$, if* $\operatorname{codh}_x \mathcal{O}_X > \dim_x \sum + 1$, *then X is normal at x.*

Here $\operatorname{codh}_x \mathcal{O}_X = \max\{k \mid \exists$ germs $f_1, \cdots, f_k$ in the maximal ideal of $\mathcal{O}_{X,x}$ such that $\forall i \leqq k$, the coset $f_i + \sum_{j<i} f_j \mathcal{O}_{X,x}$ is not a zero divisor in the ring $\mathcal{O}_{X,x}/\sum_{j<i} f_j \mathcal{O}_{X,x}\}$. For the standard concepts of sheaf cohomology with supports and their relation to the algebraic properties of the stalks the reader may consult [5], [8], [9] or [11]. This generalizes earlier results of Oka [7], Abhyankar [1], and Thimm [10] for hypersurfaces and complete intersections.

At about the same time the present writer became interested in the question of extending holomorphic differential forms across sub-