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THE NUMBER OF AUTOMORPHISMS OF AN ATOMIC BOOLEAN ALGEBRA

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A method of construction via forcing is developed which allows great freedom in the interplay among the number of atoms, number of automorphisms, size of the algebra, and such objects of settheoretic interest as c. As by-products we have

THEOREM 1. The following is consistent: there is a 0-dimensional Hausdorff space with fewer than c autohomeomorphisms, at least one of which moves a nonisolated point.

THEOREM 2. The following is consistent: there is an infinite Boolean algebra with more automorphisms than elements, the number of whose automorphisms is not a power of 2.

0. Introduction. This paper explores the interplay among the number of atoms, number of automorphisms, and size of an atomic Boolean algebra, and the relation of these cardinals to such set theoretic objects as c. As by-products we have

THEOREM 1. *The following is consistent: there is a* 0*-dimensional Hausdorff space with fewer than* c *automorphisms, at least one of which moves a nonisolated point.*

THEOREM 2. *The following is consistent: there is an infinite Boolean algebra with more automorphisms than elements, the number of whose automorphisms is not a power of* 2.

Background. De Groot and McDowell had shown that a regular Hausdorff space has uncountably many automorphisms if at least one nonisolated point is moved by some automorphism. Their proof suggested that "uncountably many" might be pushed to "at least c many," but Theorem 1 refutes this.

Theorem 2 is a partial answer to question 4 of the triple survey paper [2]. We'll say that a Boolean algebra is rich if the number of automorphisms is greater than its size. If $\lambda \leqq \kappa < 2^\lambda$ it's easy to find rich algebras of size κ with 2^λ automorphisms. Must all rich algebras be of this type? Theorem 2 says no. What cardinals are possible for the number of automorphisms of a rich algebra? Theorem 3(c), below, gives a wide range of possibilities.

The main theorem of this paper is the result of an investigation of how a construction of van Douwen could be generalized, altered,