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ON THE SPECTRUM OF CARTAN-HADAMARD MANIFOLDS

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Let M be a simply-connected complete d-dimensional Riemannian manifold of nonpositive sectional curvature K. If $K \leqq -k^2 < 0$, then the infimum of the L^2 spectrum of the negative Laplacian is greater than or equal to $(d-1)^2k^2/4$ with equality in case $K \to -k^2$ sufficiently fast at infinity. This general result is obtained by analyzing a system of ordinary differential equations. If either $d=2$ or the manifold possesses appropriate symmetry, the result is obtained under weaker conditions by analyzing a Riccati equation. Finally the case $k=0$ is treated separately.

1. Description of results. The infimum of the L^2 spectrum is defined by

$$\lambda_1 = \inf_{\phi \neq 0} \frac{\int_M |d\phi|^2}{\int_M \phi^2} \tag{1.0}$$

when the infinum is taken over H_0^1, the closure of $C_0^\infty(M)$ in the norm $\int_M (\phi^2 + |d\phi|^2)$. Let $K_x(P)$ be the sectional curvature of the two-plane $P \subseteqq M_x$, the tangent space at x. Let $\gamma(t) = \gamma(t; 0, \xi)$ be the unit-speed geodesic emanating from $0 \in M$ and having initial velocity $\xi \in M_0$. Let

$$\varepsilon(t) = \sup_{|\xi|=1} \sup_{P \subseteqq M_{\gamma(t)}} |K_{\gamma(t)}(P) + k^2|$$

where k is a positive constant. Our main result is the following upper bound.

THEOREM. *Suppose that*

$$\int_1^\infty \varepsilon(t)dt < \infty . \tag{1.1}$$

Then

$$0 < \lambda_1 \leqq (d-1)^2k^2/4 .$$

This immediately implies

COROLLARY 1. *Suppose that outside of some compact set M has constant sectional curvature $K = -k^2 < 0$. Then $0 < \lambda_1 \leqq (d-1)^2k^2/4$.*