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NOTE ON BOUNDED L^p-SOLUTIONS OF A GENERALIZED LIÉNARD EQUATION

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Two theorems are presented giving sufficient conditions for all solutions to $y''+c(t)f(y)y'+a(t)b(y)=0$ to be bounded. Furthermore, two other theorems are given guaranteeing when these solutions are L^p-solutions. Asymptotic stability is then discussed as well as several applications of these results.

In this paper, sufficient conditions will be given so that all solutions to a generalized Liénard equation,

$$y'' + c(t)f(y)y' + a(t)b(y) = 0 \tag{1}$$

will be L^p-solutions ($p \geqq 1$) on $[0, \infty)$. By an L^p-solution we shall mean a solution to (1) such that $\left(\int_0^\infty |y|^p dt\right) < \infty$. This note shall generalize some previous results (see [1] – [3]). We first need the following theorem.

THEOREM I. *Suppose $a(t)$ and $c(t)$ are continuous functions on $[0, \infty)$ and let $b(y)$ and $f(y)$ be continuous on $(-\infty, +\infty)$. Furthermore, suppose for some positive constant a_0, $a(t) \geqq a_0$, $a'(t) \leqq 0$, $c(t) \geqq 0$ for $0 \leqq t < \infty$, and $f(y) > 0$. Finally, if $B(y) = \int_0^y b(u)du \to +\infty$ as $|y| \to \infty$, then every solution of* (1) *exists on $[0, \infty)$ and $|y(t)|$, $|y'(t)|$ are bounded as $t \to \infty$.*

Proof. By standard existence theory (1) has at least one solutions satisfying $y(0) = y_0$, $y'(0) = \dot{y}_0$, and existing on some interval $[0, T)$, $T > 0$. Consider any such solutions of (1) on $[0, T)$. Multiply (1) by y' and integrate from 0 to $t < T$ to obtain,

$$\frac{1}{2}y'(t)^2 + \int_0^t c(s)f(y)y'^2 ds + \int_0^t a(s)b(y)y' ds = \frac{1}{2}\dot{y}_0^2 . \tag{2}$$

Integrating (2) by parts we have

$$\begin{aligned} &\frac{1}{2}y'(t)^2 + \int_0^t c(s)f(y)y'^2 ds + a(t)B(y(t)) \\ &\qquad - \int_0^t a'(s)B(y(s))ds = \frac{1}{2}\dot{y}_0^2 + a(0)B(y_0) \quad (0 \leqq t < T) . \end{aligned} \tag{3}$$

For $|y|$, $|y'|$ large all terms on the LHS of (3) are positive except,