PACIFIC JOURNAL OF MATHEMATICS
Vol. 94, No. 1, 1981

ULTRASPHERICAL EXPANSIONS AND PSEUDO ANALYTIC FUNCTIONS

ALLAN J. FRYANT

This paper takes up the function theoretic approach to the study of ultraspherical expansions, their conjugates, the associated elliptic equations, and first order systems. The theory of pseudo analytic functions and Bergman-Gilbert type integral operators are employed, and the relation between these two approaches is examined. Throughout, results obtained are analogs of well known theorems from the theory of analytic functions of a single complex variable, and the related study of harmonic functions and Fourier series.

The study of trigonometric series, analytic functions, Laplace's equation, and the Cauchy-Riemann system are all in a sense equivalent. Since this study has proven to be one of the most fruitful in mathematics, and since Laplace's equation is just one specific elliptic partial differential equation, analogous developments should be expected for more general elliptic equations. In particular, it is natural to hope for a relationship with analytic functions corresponding to that found in the case of harmonic functions, $u=\operatorname{Re}(f)$, which has proved so useful in the study of Laplace's equation and expansions in the associated special functions (trigonometric series).

For u the solution of an elliptic equation more general than Laplace's, two approaches are apparent:

(1) Generalize the operation of "taking the real part". That is, find bounded linear operators which transform analytic functions to solutions u. Such results have been obtained, in particular, by Bergman [3] and Gilbert [14], where *integral operators* are developed to provide the transformation from analytic functions to solutions of corresponding elliptic equations.

(2) Generalize function theory. That is, extend function theory so that solutions u of elliptic equations can be obtained as $u=\operatorname{Re}(f)$, where f is a *"pseudo" analytic function* sharing many of the properties associated with classical analytic functions of a single complex variable. Bers [4], and Vekua [24] have developed such an approach.

In the pursuit of function theoretic results in this context, the case of ultraspherical expansions is of particular interest, since in this case both approaches (1) and (2) are directly applicable and intimately related. Using them, we obtain function theoretic results for ultraspherical expansions, adding to the extensive work of Muchenhaupt and Stein [19] on the subject.