PACIFIC JOURNAL OF MATHEMATICS
Vol. 94, No. 2, 1981

SMALL DOWKER SPACES

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We construct a normal, locally compact, first countable, separable, real compact topological space which is not countabley paracompact. This construction is performed under the (relatively consistent) set-theoretic hypotheses: Martin's Axiom plus $\diamondsuit_c(E)$.

Years ago, C. Dowker proved the now well-known theorem that for any Hausdorff topological space X, topological product of X with the closed unit interval is normal iff X is both normal and countably paracompact [3]. Since then researchers have called Hausdorff spaces which are normal but *not* countably paracompact *Dowker spaces.*

Dowker spaces seem to be extremely rare and difficult to construct. In fact, in the literature there is only one which is constructed using just the usual ZFC axioms of set theory [8]. It was discovered by M. E. Rudin who then asked [10] if there were any "small" Dowker spaces, i.e., ones which are, for example, first countable, separable, realcompact, or of small cardinality.

Examples have been constructed, using extra set-theoretic axioms. M. E. Rudin [9] used the existence of a Suslin line to obtain a Dowker space which is hereditarily separable and first countable. In [5] I. Juhász, K. Kunen and M. E. Rudin construct, using CH, a first countable, hereditarily separable, realcompact Dowker space and claim that with the stronger assumption $\diamondsuit$ they could construct one which is locally compact as well.

It was unknown if Martin's axiom plus not CH allowed the construction of small Dowker spaces since Martin's axiom plus not CH often implies topological results contrary to CH or $\diamondsuit$. In this paper we construct a Dowker space using axioms of set theory consistent with Martin's axiom. This Dowker space is locally compact, first countable, separable and realcompact. By the results of [11] we cannot hope to prove that this Dowker space is hereditarily separable.

Recently, M. Bell has constructed a first countable realcompact Dowker space, assuming Martin's axiom. In fact, only a weakened form of Martin's axiom, called MA (σ-centered) is used. We use it repeatedly in our construction.

MA (*σ-centered*). If $\boldsymbol{P}$ is a partial order such that $\boldsymbol{P} = \bigcup_{n\in\omega} \boldsymbol{P}_n$ where each $\boldsymbol{P}_n$ is centered and $\kappa < 2^{\aleph_0}$ and $\{\boldsymbol{D}_\alpha : \alpha < \kappa\}$ is a collection of dense subsets o*f* $\boldsymbol{P}$, then $\boldsymbol{P}$ contains a $\{\boldsymbol{D}_\alpha : \alpha < \kappa\}$-generic subset.

Recall that a subset S of a partial order $\boldsymbol{P}$ is centered iff each