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# REPRESENTATIONS OF GAUSSIAN PROCESSES BY WIENER PROCESSES

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**Let $\{X(t),\ a \leqq t \leqq b\}$ and $\{W(t),\ 0 \leqq t < \infty\}$ be a Gaussian process and the standard Wiener process, respectively. Investigating covariance structure of $X(t)$, the paper gives various representations of $X(t)$ in terms of $W(t)$, including stochastic integral representations. Some of these representations are useful in finding hitherto unknown barrier-crossing probabilities of $X(t)$.**

1. **Introduction.** Let $X(t)$ be a Gaussian process on some interval $I$ with covariance function

$$R(s, t) = E\{X(s) - \mu(s)\}\{X(t) - \mu(t)\} ,$$

where $\mu(t)$ is the mean function, $\mu(t) = EX(t)$.

It is well-known that a Gaussian process is uniquely determined, up to the mean function, by the covariance function $R(s, t)$.

The Gaussian process which has been studied most extensively is, of course, the Wiener process $\{W(t);\ t \geqq 0\}$. Therefore, it is natural to seek representations of Gaussian processes in terms of Wiener processes.

( i ) A classical result by Doob [3, pp. 401–402] shows that: If a Gaussian process $X(t)$ has mean zero and the covariance function in the form

$$R(s, t) = u(s)v(t),\ s \leqq t$$

for $s, t$ in some interval, and if the ratio $u(t)/v(t) \equiv a(t)$ is continuous and increasing with its inverse function $a_1(t)$, then

$$X(a_1(t))/v(a_1(t)) = W(t) ,$$

where $W(t)$ is the standard Wiener process (or the Brownian motion process).

Another well-known result is the following:

(ii) If a Gaussian process $X(t)$ with zero mean has a factorable covariance function on $[0, T]^2$, i.e.,

$$R(s, t) = \int_0^T r(s, u)r(t, u)du ,$$

where for each $t \in [0, T]$ $r(t, \cdot) \in L_2[0, T]$, then $X(t)$ has a stochastic integral representation