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HOMOLOGY 3-SPHERES WHICH ADMIT NO PL INVOLUTIONS

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An infinite family of irreducible homology 3-spheres is constructed, each member of which admits no PL involutions.

1. Introduction. In Problem 3.24 of [6] H. Hilden and J. Montesinos ask whether every homology 3-sphere is the double branched covering of a knot in S^3. The interest in this question lies in the fact that there is an algorithm, due to J. Birman and H. Hilden [1], for deciding whether such a 3-manifold is homeomorphic to S^3. In addition, the Smith Conjecture for homotopy 3-spheres [4] implies that every homotopy 3-sphere of this type must be homeomorphic to S^3.

In this paper an infinite family of irreducible homology 3-spheres is exhibited which admit no PL involutions. This gives a negative answer to the above question since the nontrivial covering translation of a branched double cover is a PL involution.

2. Preliminaries. We shall work throughout in the PL category.

A *knot* K is an oriented simple closed curve in the oriented 3-sphere S^3 which does not bound a disk. The *exterior* $Q = Q(K)$ is the closure of the complement of a regular neighborhood of K. A *meridian* $\mu = \mu(K)$ of K is an oriented simple closed curve in ∂Q which bounds a disk in $S^3 - \operatorname{Int} Q$ and has linking number $+1$ with K. A *longitude* $\lambda = \lambda(K)$ of K is an oriented simple closed curve in ∂Q such that λ bounds a surface in Q and $\lambda \sim K$ in $S^3 - \operatorname{Int} Q$. ("$\sim$" means "is homologous to").

K is $\pm$ *amphicheiral* if there is an orientation reversing homeomorphism g of S^3 such that $g(K) = \pm K$. K is *invertible* if there is an orientation preserving homeomorphism g of S^3 such that $g(K) = -K$.

For the definitions of simple knot, torus knot, and fibered knot we refer to [8]. For the definitions of irreducible 3-manifold, incompressible surface, and of parallel surfaces in a 3-manifold we refer to [5]. Note that a knot K is simple if and only if every incompressible torus in $Q(K)$ is parallel to $\partial Q(K)$. If K is simple and $Q(K)$ contains an incompressible annulus which is not parallel to an annulus in $\partial Q(K)$, then K is a torus knot [3].

Suppose h is an involution on a homology 3-sphere M. Then by Smith theory [2] the fixed point set Fix $\langle h \rangle$ is homeomorphic to S^0