

\mathcal{H} -BORELIAN EMBEDDINGS AND IMAGES OF HAUSDORFF SPACES

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The following two questions are discussed in this article: (1) under what conditions is a Hausdorff space embeddable as a \mathcal{H} -Borelian subset of some Hausdorff space; and (2) under what conditions is the Hausdorff 1-1 continuous image of a \mathcal{H} -Borelian subset of a Hausdorff space a \mathcal{H} -Borelian subset of some Hausdorff space containing it. We obtain necessary and sufficient conditions in answer to question (1) in the case of a \mathcal{H}_σ and necessary conditions for \mathcal{H} -Borelian subsets of each class α if the containing space is a perfectly σ -normal \mathcal{H}_σ . Question 2 does not always have a positive answer as is shown by an example of a Hausdorff 1-1 continuous image of \mathcal{H} -Borelian subset of a compact Hausdorff space which is not \mathcal{H} -Borelian in any Hausdorff space containing it. In partial answer to question (2) necessary conditions on the domain and the range of the function are presented.

In §1 the necessary definitions are given. Many of the original definitions for completely regular spaces such as bianalytic and Borelian as well as the notion of a complete sequence of countable covers are due to Frolik and can be found in [3]. The class of σ -bianalytic spaces is an enlargement of the class of bianalytic spaces. In [8] the \mathcal{H}_σ -fiable spaces have already been introduced. For the reader's convenience, the definitions of these concepts have been given again.

The motivation of §2 is to extend to a larger class of spaces the well-known result that if $f(X)$ is a metrizable 1-1 continuous image of a \mathcal{H} -Borelian subset of a compact metric space, then $f(X)$ is a \mathcal{H} -Borelian subset of any of its metrizable compactifications. An example is given of a Hausdorff 1-1 continuous image of a \mathcal{H} -Borelian subset of a compact space which is not a \mathcal{H} -Borelian subset of any Hausdorff space in which it can be embedded. However, using some of the techniques that Frolik developed in [3] for completely regular spaces, necessary and sufficient conditions in order that a 1-1 continuous image of σ -bianalytic space X be a \mathcal{H} -Borelian subset of some Hausdorff space are obtained. This result can be applied to the cases where X is a \mathcal{H} -Borelian subset of a Hausdorff space Y and Y is either a perfectly σ -normal \mathcal{H}_σ , or has property I ([10]) or is locally metrizable.

In §3 results are presented concerning the relationship between