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A NOTE ON REGULAR CAUCHY SPACES

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A regular convergence space has both a finest and coarsest compatible regular Cauchy structure. The coarsest compatible regular Cauchy structure is complete if and only if the original space is Urysohn-closed; it is totally bounded if and only if the original space is almost topological. Minimal regular Cauchy spaces are characterized and, in the complete case, shown to be in one-to-one correspondence with the minimal regular convergence spaces. The noncomplete minimal regular regular Cauchy spaces do not have regular completions.

1. Fine and coarse Cauchy structures. The reader is asked to refer to [2] for definitions, notation, and terminology pertaining to convergence and Cauchy spaces not given here. *As in* [2], *we make the assumption that all convergence and Cauchy spaces are Hausdorff.*

A few nonstandard notations which we shall borrow from [2] are worth special mention. If filters $\mathscr{F}$ and $\mathscr{G}$ contain disjoint sets, we write "$\mathscr{F} \vee \mathscr{G} = 0$". The symbol "$\Gamma_q^n$" denotes the nth iteration of the closure operator with respect to a convergence structure q. The term "ultrafilter" will be abbreviated "u.f.".

Given a Cauchy space $(X, \mathscr{C})$, the associated convergence structure is denoted $q_{\mathscr{C}}$; in other words, $\mathscr{F}\, q_{\mathscr{C}}$-converges to x iff $\dot{x} \cap \mathscr{F} \in \mathscr{C}$.

Let $F(X)$ denote the set of all filters on X. Given a convergence space (X, q), let $[q]$ denote the set of all Cauchy structures $\mathscr{C}$ on X such that $q = q_{\mathscr{C}}$. Also associated with q are the following sets of filters:

$$\mathscr{C}^q = \{\mathscr{F} \in F(X) : \mathscr{F} \text{ is } q\text{-convergent}\}$$
$$\Lambda_q = \{\mathscr{F} \in F(X) : \mathscr{F} \vee \mathscr{G} = 0 \text{ for all } \mathscr{G} \in \mathscr{C}^q\}$$
$$\mathscr{C}_q = \mathscr{C}^q \cup \Lambda_q\,.$$

We omit the easy proof of the first proposition.

PROPOSITION 1.1. *For any convergence space (X, q), $\mathscr{C}^q$ is the finest member of* $[q]$. *$\mathscr{C}^q$ is regular* iff *q is regular. $\mathscr{C}^q$ is complete.*

PROPOSITION 1.2. *For any convergence space (X, q), $\mathscr{C}_q$ is the coarsest member of $[q]$. $\mathscr{C}_q$ is totally bounded.*

Proof. The verification that $\mathscr{C}_q$ is a Cauchy structure compatible