

ON THE WEIERSTRASS POINTS ON OPEN RIEMANN SURFACES

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The number of Weierstrass points on a compact Riemann surface of finite genus g is at most $(g - 1)g(g + 1)$ and at least $2(g + 1)$. After the Riemann-Roch's theorem for the class of canonical semi-exact differentials, Watanabe considered the number of Weierstrass points on an open Riemann surface of class O_{XD} . In this paper it will be shown that Watanabe's estimate can be proved without any conception of principal operators.

Using the notation and terminology of [1], the following theorem [6, Theorem 2] will be proved without use of the results of Mori [3], Rodin [4] and Royden [5]. Note that a meromorphic function on an open Riemann surface is said to be rational if $\text{Re } df$ is distinguished.

THEOREM. *Suppose that R is a Riemann surface of finite genus g on which $\Gamma_{h_e} \cap \Gamma_{h_{se}}^* \subset \Gamma_{h_e}^*$ holds. Then the number of Weierstrass points on R is at most $(g - 1)g(g + 1)$.*

Let S be a compact continuation of R such that the genus of S is g . Suppose P is a Weierstrass points on R and f is a rational function on R which has the only singularity of order at most g at P . Let D be a closed disk with $P \in \dot{D} \subset D \subset R$. Then the Dirichlet integral of f over $R - D$ is finite. Since $\int_{\partial D} d \text{Re } f^* = 0$, there exist harmonic functions u on $S - D$ and v on R such that $u - v = \text{Re } f$ on $R - D$. Thus we have $dv \in \Gamma_{h_e} \cap \Gamma_h^*$.

We wish to show that dv^* is semi-exact on R . If c is a dividing cycle on $R - D$, then c is homologous to zero on $S - D$. This gives that

$$\int_c dv^* = \int_c du^* - \int_c d \text{Im } f = 0.$$

Since $dv \in \Gamma_{h_e} \cap \Gamma_{h_{se}}^*$, it follows from the assumption that $dv^* \in \Gamma_{h_e}$.

We define

$$\lambda = \begin{cases} du & \text{on } S - D \\ dv + d \text{Re } f & \text{on } R. \end{cases}$$

Then λ and λ^* have no periods along any cycle b on S , where $b \not\ni P$. Therefore $\int \lambda + i\lambda^*$ is a meromorphic function on S . It is easy to