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COMPLETELY REGULAR ABSOLUTES AND PROJECTIVE OBJECTS

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The absolute (EX, π_X) is constructed for an arbitrary space X and is shown to be unique with respect to EX being extremally disconnected and completely regular and π_X being a θ-continuous, perfect, separating irreducible surjection. A function $f: X \to Y$ is said to have a continuous E-lifting if there is a continuous function $F: EX \to EY$ such that $\pi_Y \circ F = f \circ \pi_X$. A class of functions, called η-continuous, is introduced, shown to contain the class of continuous functions and the class of θ-continuous, closed surjections, and proved to have continuous E-liftings. Functions which have continuous E-liftings are completely characterized as being the composition of η-continuous functions.

1. Introduction and preliminaries. In 1963, Iliadis (see [7]) constructed, for a Hausdorff space X, an extremally disconnected Tychonoff space EX and an irreducible, perfect θ-continuous surjection $\pi_X: EX \to X$ and showed that (EX, π_X) is unique in this sense: If Y is an extremally disconnected, Tychonoff space and $f: Y \to X$ is an irreducible, perfect, θ-continuous surjection, then there is a homeomorphism $g: EX \to Y$ such that $f \circ g = \pi_X$. In 1969, Mioduszewski and Rudolf [9] modified this construction to obtain a space aX which has the same underlying set as EX and the topology of aX is generated by the topology of EX plus $\{\pi_X^{-1}(U): U \text{ open in } X\}$. The function $a_X: aX \to X$ is the same as the function π_X. The space aX is extremally disconnected and Hausdorff, and the function a_X is an irreducible, perfect continuous surjection. Also, (aX, a_X) is shown to be unique in the sense similar to the uniqueness of (EX, π_X). So, there is a trade-off — the Tychonoffness of EX is reduced to Hausdorff for aX, but the θ-continuity of π_X is strengthened to continuity for a_X. Both EX and aX are called *absolutes* of X.

More recently, Sapiro [11] and Ul'janov [13] extended the construction and uniqueness of (aX, a_X) (aX is denoted in [11] by qX) for an arbitrary topological space X. In this case, aX is extremally disconnected and a_X is a separating, irreducible perfect continuous surjection. Also, they showed if $f: X \to Y$ is a continuous function between spaces X and Y, there is a continuous function $F: aX \to aY$ such that $a_Y \circ F = f \circ a_X$.

In the second section of this paper we characterize the projective objects in the category of spaces and perfect separating continuous functions as morphisms. As a cosequence, we obtain the