PACIFIC JOURNAL OF MATHEMATICS
Vol. 95, No. 1, 1981

ON ISOMETRIES OF HARDY SPACES ON COMPACT ABELIAN GROUPS

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Let $H^p(m)$, $0 < p \leqq \infty$, be the Hardy spaces on a quotient K of the Bohr group. In this paper we completely determine the isometries of $H^p(m)$, $p \neq 2$, onto itself. Our result is a generalization of a recent work of Muhly who determined the isometries of $H^p(m)$ onto itself under the assumption that the dual group of K is countable, and it may be regarded as a partial answer to a question posed by Muhly.

1. Introduction. Many results have been obtained concerning isometries of Hardy spaces in the theory of uniform algebras. The most fundamental result in this direction is due to de Leeuw, Rudin, and Wermer [2], which states that an automorphism of the classical Hardy space $H^\infty(\boldsymbol{T})$ is induced via composition with the unit circle T of a fractional linear transformation of the unit disc onto itself. Their work was carried on independent of Nagasawa [13], who also described the isometries of $H^\infty(\boldsymbol{T})$ onto itself. On the other hand, Arens [1] completely determined the automorphisms of the uniform algebra of analytic functions on a compact abelian group K whose dual group Γ is archimedean ordered (cf. [11]). This result was extended by Muhly [11] to the uniform algebra of analytic functions induced by a flow which has no periodic orbits. Moreover Muhly [12] has recently given, among other things, the following interesting generalization of this result of Arens to the case of isometries of Hardy spaces $H^p(m)$, $p \neq 2$, on K: Under the assumption that Γ is countable, every isometry of $H^p(m)$, $p \neq 2$, is induced via composition with an affine map of K such that the adjoint of the additive factor of this map preserves the order of Γ. The purpose of this paper is to remove the assumption on Γ. This result provide a partial positive answer to the following question posed by Muhly in [12; §5]:

Is it possible to describe the isometries of ergodic Hardy spaces onto itself without the separability assumptions on phase spaces?

The difficulty is that, in the absence of separability assumptions automorphisms of measure algebras may not have point realizations. On the other hand, although our proof rests on some techniques which were first investigated by Muhly [11], [12], and is given in the context of almost periodic setting, one will find some improvements of the proof given in [12; §3].