PACIFIC JOURNAL OF MATHEMATICS
Vol. 95, No. 2, 1981

LOCATED SETS ON THE LINE

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Located sets are sets from which the distance of any point may be measured; they are used extensively in modern constructive analysis. Here a general method is given for the construction of all located sets on the line. It is based on a characterization of a located set in terms of the resolution of its metric complement into a union of disjoint open intervals. The characterization depends on a strong countability condition for the intervals, called the *locating condition*. Included as a special case is the characterization and construction of compact sets. The techniques used are in accord with the principles of Bishop's *Foundations of Constructive Analysis*, 1967.

In many situations it is desired to measure the distance

$$\rho(x, G) \equiv \inf \{\rho(x, y) : y \in G\}$$

between a point x and a set G in a metric space. However, this is *not* always possible constructively. By this we mean that a counterexample exists in the sense of Brouwer; discussions of these are found in [1] (and [4]). The italicized word "*not*" will also be used below in this sense.

Brouwer [2] introduced the concept of *located* set, for which the above distances always exist. Here the concept of located set on the line is reduced to the concept of number. The construction of an arbitrary located set is reduced to the construction of two sequences of real numbers with certain properties.

The metric complement of a located set G is the set

$$-G \equiv \{x : \rho(x, G) > 0\} .$$

Such a set is said to be *colocated*.

The characterization of a located set G on the line is obtained by means of the resolution of its metric complement $-G$ into a countable union $\bigcup_n I_n$ of disjoint open intervals, given in [4]. It is shown in [3] that only the closure of a located set G may be recovered from its metric complement $-G$. Thus we characterize *closed* located sets. Arbitrary located sets are precisely the dense subsets of these.

The characterization theorem below involves four conditions on the sequence $\{I_n\}$ of open intervals. Briefly, these are the following.

(1) The intervals are *fixative*, i.e., each is either void or *fixed*