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# SMOOTH ACTIONS OF THE CIRCLE GROUP ON EXOTIC SPHERES

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**Recent work of Schultz translates the question of which exotic spheres $S^n$ admit semifree circle actions with $k$-dimensional fixed point set entirely to problems in homotopy theory provided the spheres bound spin manifolds. In this article we study circle actions on homotopy spheres not bounding spin manifolds and prove, in particular, that the spin boundary hypothesis can be dropped if $(n-k)$ is not divisible by 128. It is also proved that any ordinary sphere can be realized as the fixed point set of such a circle action on a homotopy sphere which is not a spin boundary; some of these actions are not necessarily semi-free. This extends earlier results obtained by Bredon and Schultz. The Adams conjecture, its consequences regarding splittings of certain classifying spaces and standard results of simply-connected surgery are used to construct the actions. The computations involved relate to showing that certain surgery obstructions vanish.**

**1. Introduction.** Results due to Schultz give a purely homotopy theoretic characterization of those homotopy $(n + 2k)$-spheres admitting smooth semi-free circle actions with $n$-dimensional fixed point sets provided one limits attention to exotic spheres bounding spin manifolds. The method of proof is similar to that described in [14] for actions of prime order cyclic groups; a detailed account will appear in [21]. Since the premise of this article relates directly to [21], we outline some of the results contained there.

Given nonnegative integers $m < n$, let $CP^n_m$ denote the quotient complex $CP^n/CP^m$. $CP^n_m$ is also the Thom space of $m$ copies of the canonical line bundle over $CP^{n-m}$ [10, 11]. For some integer $A$ depending only on $n - m$, the complexes $\sum^A CP^n_m$ and $CP^{n+A}_{m+A}$ are stably equivalent [4]. Using this periodicity one can define a spectrum (finite) $CP^n_m$ for all integers $m < n$. In the same manner, it is also possible to form a limit spectrum $CP^\infty_m$ for all finite integers $m$. If $m$ and $n$ are positive integers, there are elementary coexact sequences of the form

$$S^{2m-2} \longrightarrow CP^n_{m-1} \longrightarrow CP^n_m \xrightarrow{\delta} S^{2m-1} ;$$

and by periodicity and limit arguments there are similar sequences when $m$ and $n$ are arbitrary integers or $n = \infty$.

Let $F_{S^1}(C^q)$ be the topological monoid of $S^1$ equivariant self-maps