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A NOTE ON FR-PERFECT MODULES

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This paper defines and characterizes projective FR-perfect modules which is a generalization of semiperfect modules. Using these some characterizations of semiperfect modules are obtained.

Introduction. Let R be a ring with identity. All modules we consider are unitary left R-modules. A submodule $N \subseteqq M$ is said to be *small* in M if $N + T = M$ implies $T = M$. An epimorphism $P \xrightarrow{f} M$ is called *minimal* if $\text{Ker}(f)$ is small in P. A minimal epimorphism $P \xrightarrow{f} M$, where P is projective, is called a *projective cover* of M. We denote by J, the Jacobson radical of R. By $J(M)$ we mean the radical (intersection of all maximal submodules) of a module M. If M is projective $J(M) = JM$. We call a module N *M-finitely related* (M-FR) if $N \cong M^n/B$, where B is finitely generated. A module M is called FR-*perfect* if every M-FR module has a projective cover. Similarly we define finitely presented perfect (FR-perfect) modules.

Our aim is to characterise M-FR perfect projective modules. So we would like to find out equivalent conditions for a module M/A to have a projective cover, where M is projective. In §1 we do this when either A is finitely generated or JM is small in M. In particular, we show that for a projective module M, M/A has a projective cover if and only if $f(A)$ is a direct summand of M/JM and any direct decomposition $f(A) \oplus B$ of M/JM can be lifted up, where the summand of M corresponding to $f(A)$ is finitely generated and $f: M \rightarrow M/JM$ is the natural projection. So for a projective FR-perfect module every finitely generated submodule of M/JM is a direct summand. That is M/JM is a regular R/J-module (R. Ware [8]). We prove some properties of regular modules which are used later in proving that direct sum of FR-perfect projective modules is FR-perfect projective if and only if each summand is so.

In §2, we give several characterizations of FR-perfect projective modules. We prove that the following conditions are equivalent for a projective R-module M (i) M is FR-perfect (ii) M/JM is a regular R/J module and any direct decomposition $A \oplus B$ of M/JM can be lifted up whenever A is cyclic (finitely generated) and the summand of M corresponding to A is finitely generated and (iii) M/U has a projective cover whenever U is cyclic (finitely generated). If further JM is small in M then the above conditions are equivalent to (iv)