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PARTITION NUMBERS FOR TREES AND ORDERED SETS

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In this paper some bounds on the Tveberg-type convexity partition numbers of abstract spaces will be presented. The main objective is to show that a conjecture of J. Eckhoff relating the Tverberg numbers to the Radon number is valid for a certain class of spaces which include ordered sets, trees, pairwise products of trees and subspaces of these. (Application of the Main Theorem to a certain class of semilattices is given in an appendix.) For ordered sets the results here improve those of P. W. Bean and are best possible for general ordered sets.

1. Introduction. To establish terminology, recall that an *alignment* [7, 8] ("algebraic closure system" [2]) on a set X is a family $\mathscr{L}$ of subsets of X—to be regarded as "convex" subsets—such that

(A1) $\varnothing, X \in \mathscr{L}$,

(A2) arbitrary intersections of sets in $\mathscr{L}$ are again in $\mathscr{L}$,

(A3) unions of upward directed families of sets in $\mathscr{L}$ are again in $\mathscr{L}$.

The smallest convex set containing a set S is denoted $\mathscr{L}(S)$ and is called the *hull of* S.

A *Radon m-partition* of S is a partition of S into m subsets

$$S = A_1 \cup \cdots \cup A_m$$

such that

$$\mathscr{L}(A_1) \cap \cdots \cap \mathscr{L}(A_m) \neq \varnothing .$$

Any point in such an intersection will be called an *m-partition point* of S. It is desirable to allow repeated points in S. This can be formalized by letting S be a multiset: each point of S has an integral multiplicity which determines the maximum number of A_i's in which it may be used. (The *cardinality* $|S|$ of a multiset S is then the sum of the multiplicities of its points.) Sometimes it will be more convenient to think of S as indexed by some set of $|S|$ distinct indices and to associate partitions of the index set with partitions of S.

The mth *partition number* $p_m(X)$ of an aligned space X is the smallest integer (if such exists) such that any multiset of cardinality p_m of points from X admits a Radon m-partition. (The smallest integer $\bar{p}_m$ such that any set of $\bar{p}_m$ *distinct* points admits a Radon m-partition will be called the *restricted* mth *partition number*.)