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A SIMPLE GENUS ONE KNOT WITH INCOMPRESSIBLE SPANNING SURFACES OF ARBITRARILY HIGH GENUS

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The following theorem is proved. There exists a simple genus one knot with incompressible spanning surfaces of arbitrarily high genus.

1. Introduction. H. C. Lyon in [2] proved that there exists a genus one knot which has incompressible spanning surfaces of arbitrarily high genus. Lyon's knot has companions and the companions are essential to his discussion. Presented in this paper is a knot of genus one which is shown to have no companions (is simple) but which has incompressible spanning surfaces of arbitrarily high genus. The discussion is in the PL category and all knots and surfaces are tamely embedded in S^3. The notation and terminology generally follow that of [2], [3], [4], [1]. All surfaces are nonsingular unless otherwise indicated. When two surfaces are discussed it will be assumed that they are in general position so that their intersection consists of at most disjoint simple closed curves (scc) and spanning arcs.

2. The example. The knot K is shown in Figure 1. Figure 2 shows a singular disc D^* bounded by K. Figure 3 shows D^* with the singularities removed by cutting out two discs D', D'' from D^*. In Figure 3 an annulus H has been attached to the "hatched" side of D^* along the two boundaries $\partial D', \partial D''$. The annular tube H surrounds a part of D^*. Thus Figure 3 shows an orientable surface $S(-1)$ of genus one spanned by K, therefore K has genus at most one.

3. Some preliminaries. Ball (3-cell) $\tilde{Q}$ is selected as in Figure 4. Sphere (2-sphere) $C = \partial\tilde{Q}$ contains two disjoint subdiscs $M(1)$ and $M(4)$ which contain the points $K \cap C$ in two special classes. $\tilde{Q}$ is subdivided into three subballs $Q(1)$, $Q(2)$, $Q(3)$ by two subdiscs $M(2)$ and $M(3)$ as illustrated in Figure 4 so that each sphere $C(i) = \partial(Q(i))$, $i = 1, 2, 3$, contains two discs $M(i)$ and $M(i+1)$ which have the points of $K \cup C(i)$.

Figure 5 illustrates $K \cap Q(i)$, $i = 1, 2, 3$. There are four simple arcs $a(i, 1)$, $a(i, 2)$, $b(i, 1)$, $b(i, 2)$ of $K \cap Q(i)$. Figure 6 shows how each of these arcs is completed by one of four disjoint simple arcs $\alpha(i, 1)$, $\alpha(i, 2)$, $\beta(i, 1)$, $\beta(i, 2)$ in $C(i)$ so that each $b(i, j) \cup \beta(i, j)$, $j =$