PACIFIC JOURNAL OF MATHEMATICS
Vol. 96, No. 1, 1981

SOME HOMOLOGY LENS SPACES WHICH BOUND RATIONAL HOMOLOGY BALLS

ANDREW J. CASSON AND JOHN L. HARER

A homology lens space is a smooth closed 3-manifold M^3 with $H_k(M^3)=H_k(L(p,1))$ for all k (p some nonnegative integer). When $p=1$ M^3 is a homology 3-sphere. It is an open question which of these homology lens spaces bound rational homology balls and of special interest which homology 3-spheres bound contractible manifolds. In this note we answer this question for certain Seifert fibre spaces, each with three exceptional fibres.

Let p, q, r and d be integers. Whenever l, m, and n can be defied by the relation $lqr + mpr + npq = d$, there is a well-defined orientable Seifert fibre space $M^3(p, q, r; d)$ with three exceptional fibres of type (p, l), (q, m) and (r, n). When $d = 1$ and p, q, r are coprime and positive, M^3 is the Brieskorn manifold $\Sigma(p, q, r) = \{(x, y, z) \in C^3\colon\ x^p + y^q + z^r = 0;\ x\bar{x} + y\bar{y} + z\bar{z} = 1\}$; and when $p = 1$ (so that the corresponding fibre is no longer exceptional) M^3 is a genuine lens space.

THEOREM. *Let $(p, q, r; d)$ be in one of the following six classes:*
(1) $(p, ps \pm k, ps \pm 2k; k^2)$ *for p odd*
(2) $(p, ps - k, ps + k;\ k^2)$ *for p even and s odd*
(3) $(p, q, s^2(p + q - pq) + s(2p - pq) + p; (s(p + q - pq) + p)^2)$
(4) $(p, st + s + 1, pt(s + 1) - (st + s + 1); (pst - (st + s + 1))^2)$
(5) $(p, p, 1 - s; p^2s^2)$
(6) $(p, p, 4s + 1; 4p^2s^2)$.
If M^3 is the associated Seifert fibre space, M^3 can be realized as the boundary of a Mazur-type manifold obtained by adding a 2-handle to $S^1 \times B^3$. In particular, the Brieskorn homology spheres which arise when $d = 1$ bound contractible 4-manifolds.

These Brieskorn classes include $\Sigma(2, 3, 13)$, $\Sigma(2, 5, 7)$, and $\Sigma(3, 4, 5)$ which are shown to bound Mazur manifolds in (1). Along the way we recover also the fact that the lens spaces $L(t^2, qt + 1)$, for q and t relatively prime, bound Mazur-type 4-manifolds of the kind mentioned above.

At present we have three methods of constructing these 4-manifolds. We will sketch two of them and prove the theorem with the third. We wish to thank A. Goalby and P. Melvin for helpful discussions.