PACIFIC JOURNAL OF MATHEMATICS
Vol. 96, No. 2, 1981

A CHARACTERIZATION OF THE ADJOINT L-KERNEL OF SZEGÖ TYPE

SABUROU SAITOH

Let G be a bounded regular region in the complex plane and $\hat{L}(z, u)$ the adjoint L-kernel of Szegö kernel function $\hat{K}(z, \bar{u})$ on G. Then, for any analytic function $h(z)$ on G with a finite Dirichlet integral, it is shown that the equation

$$\frac{1}{\pi}\iint_G |h'(z)|^2 dxdy = \int_{\partial G}\int_{\partial G} |(h(z_1)-h(z_2))\hat{L}(z_1, z_2)|^2 \, |dz_1| \, |dz_2|$$

holds. Furthermore, for any fixed nonconstant $h(z)$, we show that the function $\hat{L}(z_1, z_2)$ on $G\times G$ is characterized by that equation in some class.

1. Introduction and statement of result. Let S denote an arbitrary compact bordered Riemann surface. Let $W(z, t)$ be a meromorphic function whose real part is the Green's function $g(z, t)$ with pole at $t \in S$. The differential id $W(z, t)$ is positive along ∂S. For simplicity, we do not distinguish between points $z \in S \cup \partial S$ and local parameters z. For an arbitrary integer q and for any positive continuous function $\rho(z)$ on ∂S, let $H^q_{p,\rho}(S)[p \geqq 1]$ be the Banach space of analytic differentials $f(z)(dz)^q$ on S of order q with finite norms

$$\left\{\frac{1}{2\pi}\int_{\partial S} |f(z)(dz)^q|^p \rho(z)[\mathrm{id}\ W(z, t)]^{1-pq}\right\}^{1/p} < \infty ,$$

where $f(z)$ means the Fatou boundary value of f at $z \in \partial S$. Let $K_{q,t,\rho}(z, \bar{u})(dz)^q$ be the reproducing kernel for $H^q_{2,\rho}(S)$ which is characterized by the reproducing property

$$f(u) = \frac{1}{2\pi}\int_{\partial S} f(z)(dz)^q \overline{K_{q,t,\rho}(z, \bar{u})(dz)^q}\rho(z)[\mathrm{id}\ W(z, t)]^{1-2q}$$

$$\text{for all } f(z)(dz)^q \in H^q_{2,\rho}(S) .$$

See [9]. Let $L_{q,t,\rho}(z, u)(dz)^{1-q}$ denote the adjoint L-kernel of $K_{q,t,\rho}(z, \bar{u})(dz)^q$. The function $L_{q,t,\rho}(z, u)(dz)^{1-q}$ is a meromorphic differential on S of order $1 - q$ with a simple pole at u having residue 1. Moreover,

$$\overline{K_{q,t,\rho}(z, \bar{u})(dz)^q}\rho(z)[\mathrm{id}\ W(z, t)]^{1-2q} = \frac{1}{i}L_{q,t,\rho}(z, u)(dz)^{1-q} \text{ along } \partial S . \tag{1.1}$$